Full Extractors for Logical Processing in Hypergraph Product Codes

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Background

- **Goal:** Low-overhead fault-tolerant quantum computation (FTQC) with QLDPC codes and fixed connectivity.
- **Extractor architectures (EAs):** Ancilla systems called *extractors* measure logical Paulis via generalized code surgery ⇒ universal FTQC via Pauli-based computation (PBC).
- **Three resource tradeoffs:** space (extra qubits), compilation overhead (native measurements per logical Pauli), connectivity (max qubit degree).
- A **full** extractor measures *every* logical Pauli on a block.

Comparison with Existing Work

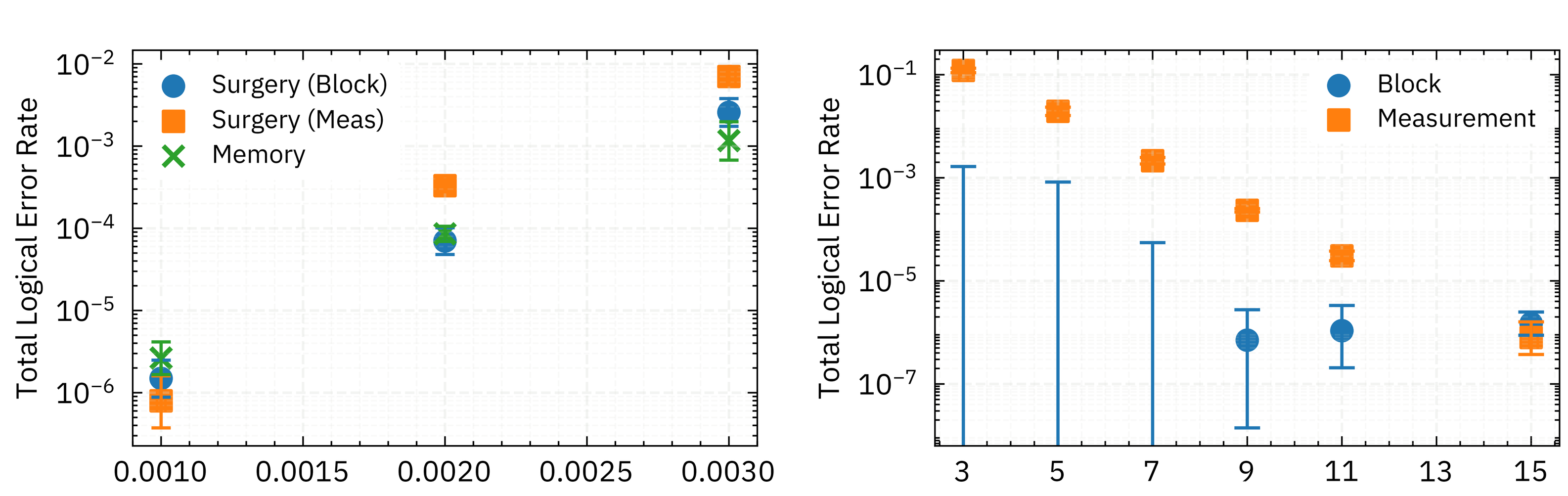
Ref.	Code Family	d	k	n_{tot}	$\frac{k}{n_{\text{tot}}}$	Compilation Overhead	Max Degree
[Lit19]	Surface (Fast)	11	1	482	0.21%	1×	4
		17	1	1154	0.09%		
	Surface (Compact)	11	1	362	0.28%	≤ 8×	
		17	1	866	0.12%		
[Yod+25]	BB	12	12	338	3.55%	≤ 25×	7
		18	12	734	1.63%		
This work	HGP	8	32	1605	1.99%	1×	10
		10	50	3011	1.66%		
		16	50	5651	0.88%		
[Web+26]	GB	6	10	244	4.10%	1×	Reconf.
		10	12	452	2.65%		
		16	14	860	1.63%		
[Cai+26]	BB	≤ 18	10	875	1.14%	1×	Reconf.
	LP	≤ 20	148	3742	3.96%		

Acronyms: BB: bivariate bicycle; GB: generalized bicycle; LP: lifted product. n_{tot} = data + check qubits in the code block *and* extractor. "Reconf." = reconfigurable connectivity.

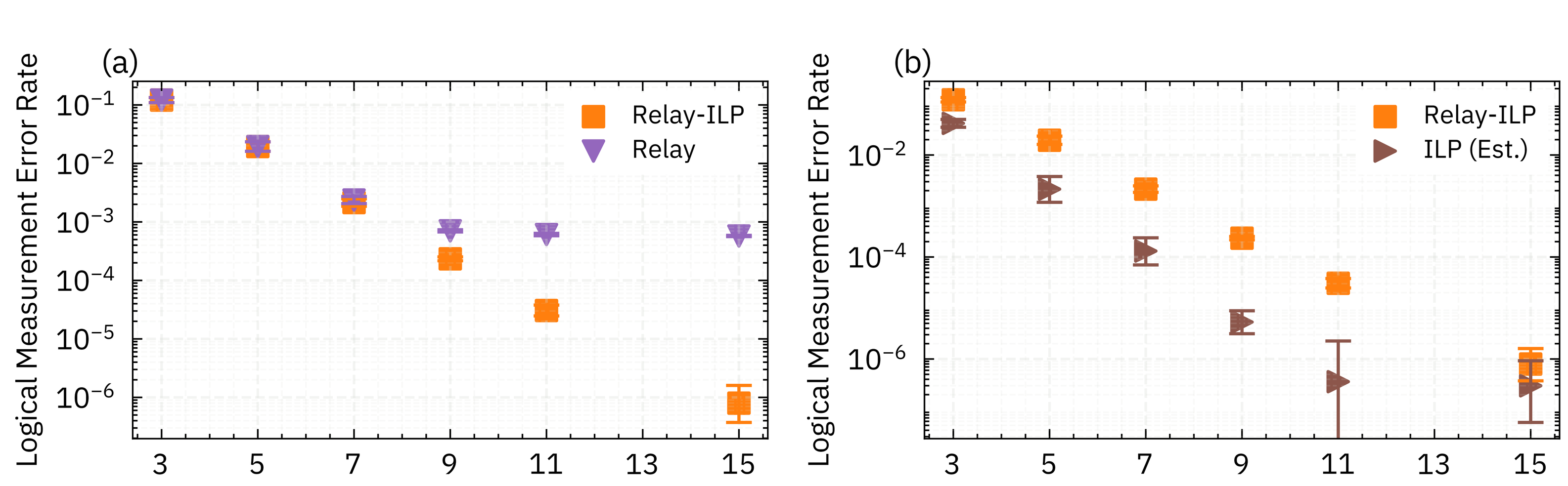
Circuit-Level Noise Simulations

Setup:

- Code: $[[882, 50, 10]]$ HGP; measure a random logical operator (logical weight 37, physical weight 197).
- Noise: circuit-level depolarizing at rate p (idle $p/10$).
- Decoder: **two-stage** – Relay-BP [Mül+25] → ILP fallback.



Logical error rates of the $[[882, 50, 10]]$ **HGP + extractor.** "Meas" = measurement; "Block" = other 49 qubits.



Decoder comparison. Two-stage Relay-BP + ILP is essential.

Key result. At $p = 0.1\%$, $N_R = 15$:

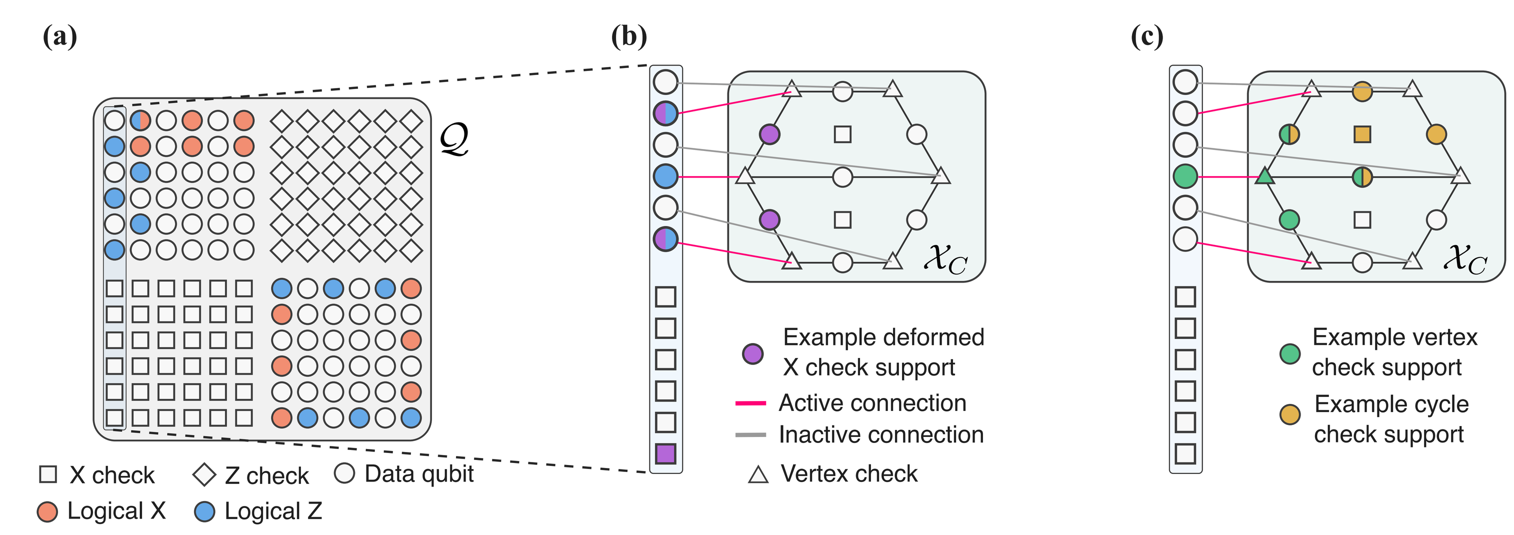
- logical measurement error $\sim 10^{-6}$,
- block error similar to base-code memory.

Construction

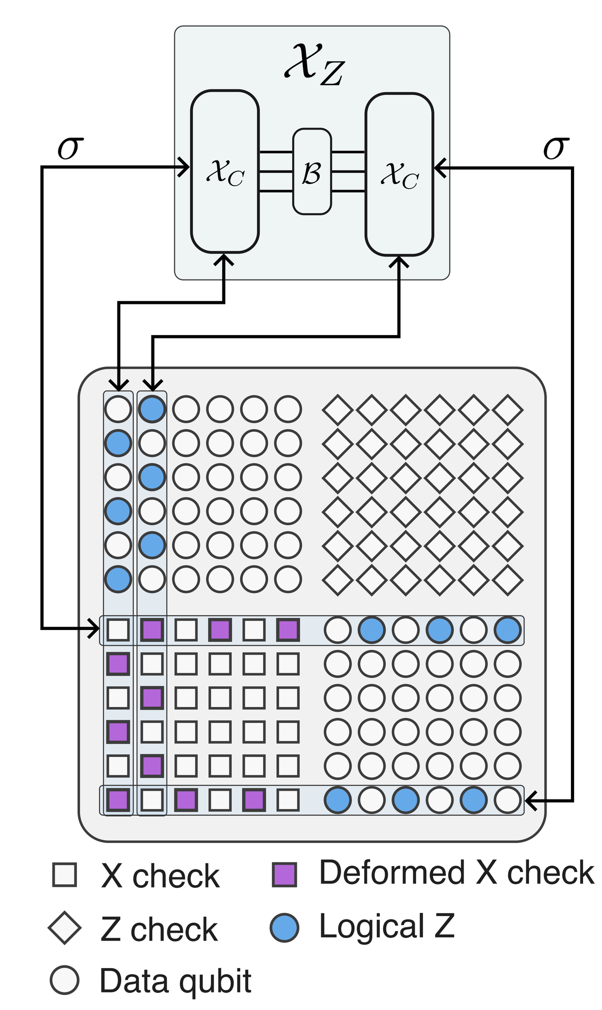
Base codes: cyclic HGP codes [ADT25].

Three-step construction:

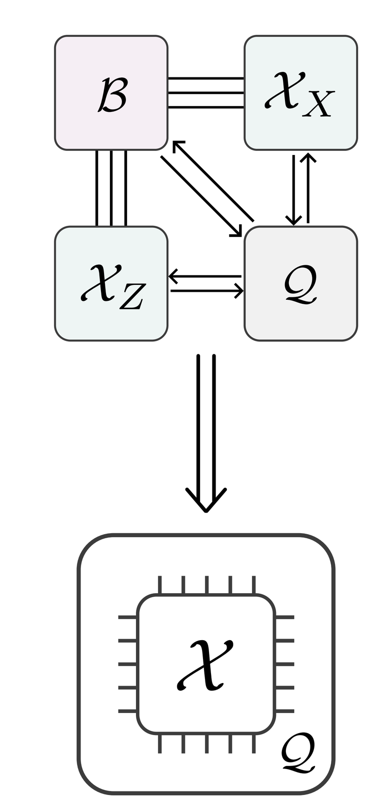
1. **Single-column extractor.** Search for a graph $G = (V, E)$, $|V| = n$, whose extractor is *distance-preserving*.



2. Z / X **basis extractors.** Bridge k single-column extractors ⇒ measures any logical Z (X).
3. **Full extractor.** Bridge information vertices of $\mathcal{X}_X$ and $\mathcal{X}_Z$.



Single-basis extractor.



Full extractor from $\mathcal{X}_X$, $\mathcal{X}_Z$, and bridge $\mathcal{B}$.

Theorem (informal). For a $[[2n^2, 2k^2, d]]$ cyclic HGP code with a distance-preserving single-column extractor on $G = (V, E)$, there exists a **full extractor** using $\Theta(k(|E| + |V| + k))$ extra qubits/checks. Any logical operator can be measured with phenomenological fault distance

$$\min(d,\ k^2,\ k \cdot \lambda(G)),$$

where $\lambda(G)$ is the edge connectivity of G.

Conclusions

- Our extractors simultaneously achieve:
  - arbitrary logical Pauli measurements,
  - fixed max-degree-10 connectivity,
  - 1.5–1.8 space overhead compared to the base code,
  - $\sim 10^{-6}$ logical measurement error at $p = 0.1\%$.
- Fixed-connectivity platforms can thus capture the QLDPC rate advantage without a compilation-overhead penalty vs. surface codes.

Open questions:

- Real-time decoding
- Improved syndrome measurement circuits
- Extensions to other code families

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