Parsimonious Quantum Low-Density Parity-Check Code Surgery

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Abstract

Quantum code surgery is useful for logical measurements of quantum error-correcting codes. It includes several fault-tolerant logical computation schemes, e.g., Table 1.
The efficiency of these schemes depends on constructing low-overhead ancilla systems for quantum Low-Density Parity-Check (qLDPC) codes. **In this work, we construct an ancilla system of qubit size $O(W \log W)$ to measure an arbitrary logical Pauli operator of weight W in any qLDPC stabilizer code.**
This new construction immediately reduces the asymptotic overhead across various quantum code surgery schemes.

Main Result

Space overhead	Previous best	This work
Single measurement	$O(W \log^3 W)$ [6]	$O(W \log W)$
Universal adapters	$O(tW \log^3 W)$ [5]	$O(tW \log W)$
Parallel measurement	$O(tW(\log t + \log^3 W))$ [3]	$O(tW(\log t + \log W))$
Extractors	$O(n \log^3 n)$ [4]	$O(n \log n)$
Single-shot surgery	$O(nkd(\log k + \log^3 n))$ [1]	$O(nkd(\log k + \log n))$
Constant-time surgery	$O(n \log^3 n)$ [2]	$O(n \log n)$

Table 1:Comparison of the space overheads of different fault-tolerant logical measurement schemes on qLDPC codes, prior to this work and after including the construction in this work.

In Table 1,

- t is the number of logical Pauli operators being measured
- W is the maximum weight of the logical representatives being measured
- n, k, d are the system size, quantum dimension and code distance of the qLDPC code

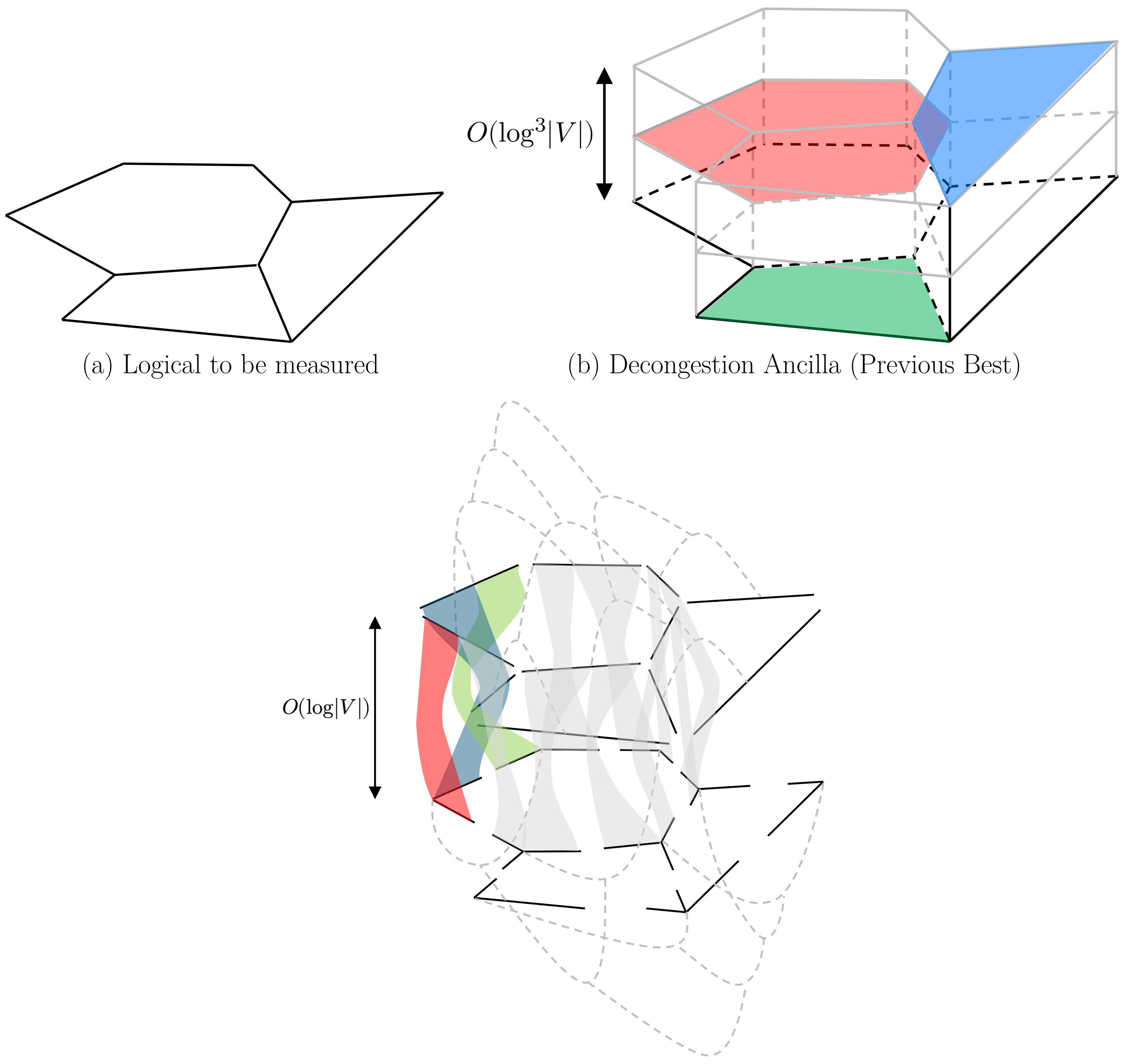
For adapters, it is presumed that t different sparsely overlapping logical operators are measured jointly. The first four gadgets are applicable to all stabilizer codes.

Theorem (Formal Result)

Let $G = (E, V)$ denote a graph with bounded degree (e.g., Fig. 1a). Then there exists cell complex A with faces F^A, edges E^A, vertices V^A (see Fig. 1c) such that

- All faces are adjacent to bounded number of edges (and vice-versa);
- All cycles of A are generated by boundaries of faces in F^A;
- (E^A, V^A) is a connected graph of bounded degree containing G as a subgraph;
- $|V^A| = O(|V| \log |V|)$

Ancilla Cartoon



(a) Logical to be measured

(b) Decongestion Ancilla (Previous Best)

(c) Parsimonious Ancilla (This Work)

Figure 1:Parsimonious Ancilla System. (a) An example of a measurement graph of a logical operator. (b) The (previous best) ancilla obtained via the decongestion lemma so that cycles are now contractible due to the faces (colored) and the congestion is controlled. (c) The parsimonious ancilla obtained in this work.

In Fig 1c two binary trees are constructed, based on the *local structure* of edges (top) and vertices (bottom). *Shuffles* of $O(\log |V|)$ binary trees induce faces (depicted in color and grey) so that the local views are paired up to reveal the *global structure*.
In particular, any cycle in 1a is embedded in 1c by traversing between the top and bottom binary trees so that the boundaries of the shuffling faces correspond to a cycle in 1a (e.g., colored and grey). Hence, cycles in 1a are products of local, low-density faces in 1c.

References

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