Logical Computation on QLDPC Codes through Surgery

With Case Study on Bivariate Bicycle codes

Andrew Cross, **Zhiyang He (Sunny)**, Tomas Jochym-O'Connor, Patrick Rall, **Esha Swaroop**, Dominic Williamson, Theodore Yoder

Outline

- Background and Motivation
    - Quantum LDPC Codes
    - Code Surgery Methods
- Auxiliary Graph Surgery on QLDPC Codes
    - Graph Desiderata
    - Universal Adapter for Joint-measurements
- Case Study: [[144,12,12]] Bivariate bicyclic code

Basics of QEC

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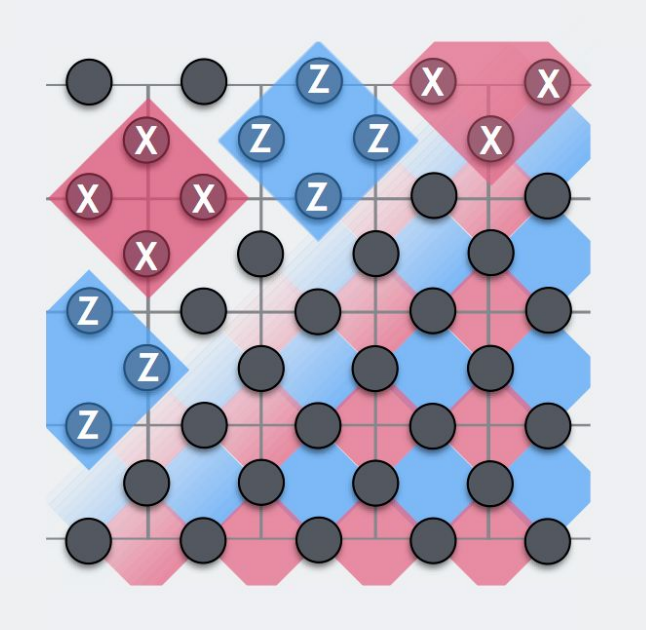
One of the most promising codes: Surface Code.

1. Built on a 2D lattice of qubits.
2. Parameters [n= $2L^2$, k = 1, d = L].
3. Experimental demonstration of subthreshold scaling by Google*.

Challenge:

1. Significant space overhead (~1000x)!



* Quantum error correction below the surface code threshold [Google and collaborators (2024)]

** Surface code figure credit to Niel de Beaudrap.

Quantum LDPC Codes: the Tradeoff

Quantum Low-Density Parity-Check (LDPC) Codes:
stabilizers of $O(1)$ weight, qubits in $O(1)$ stabilizers.
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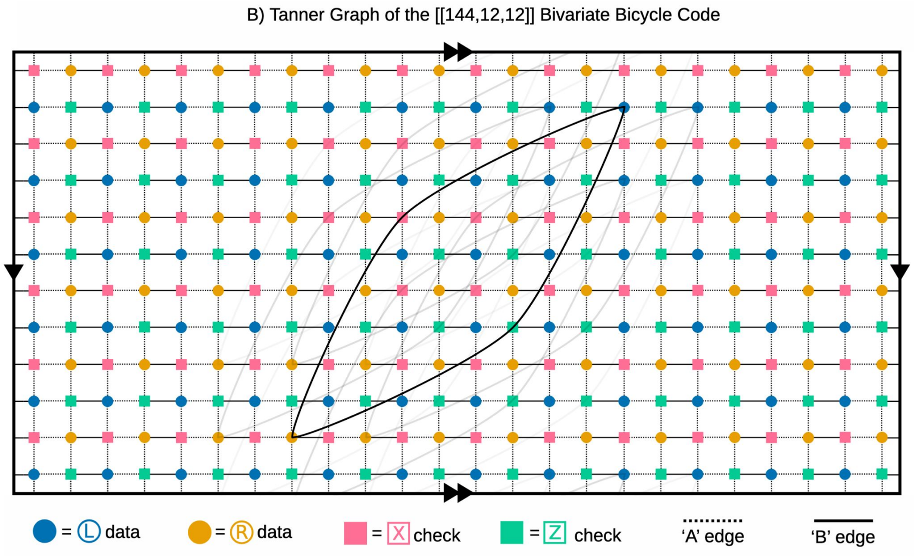
Practically: many codes with nice parameters, such as the quasi-cyclic lifted product codes and IBM's [n = 144, k = 12, d = 12] Bivariate Bicycle Code.*

Theoretically: Asymptotically good codes with k, d = O(n).**



* High-threshold and low-overhead fault-tolerant quantum memory [Bravyi et al 2023]
** Asymptotically Good Quantum and Locally Testable Classical LDPC Codes [Panteleev Kalachev 2021]

The Challenge: Logical Computation on Quantum LDPC Codes

QLDPC codes could be >10x more space-efficient than surface code. However,

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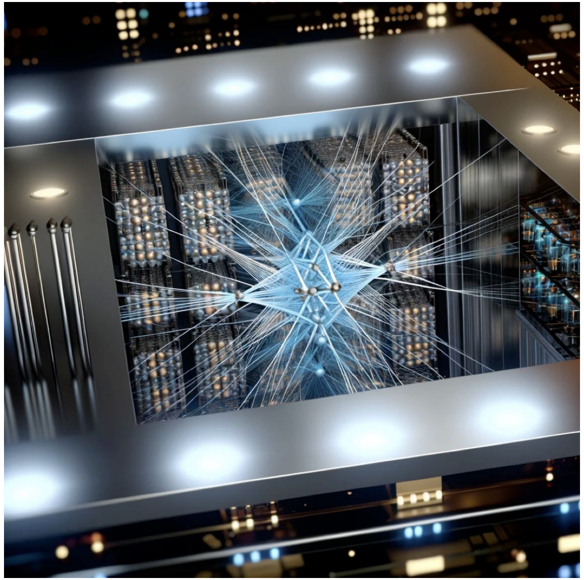

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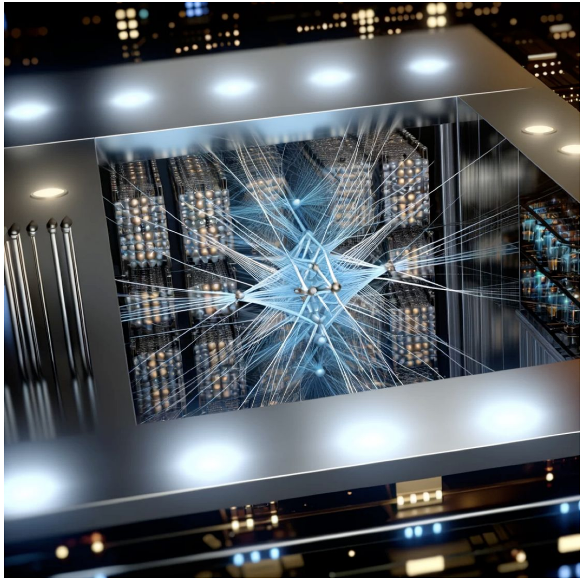

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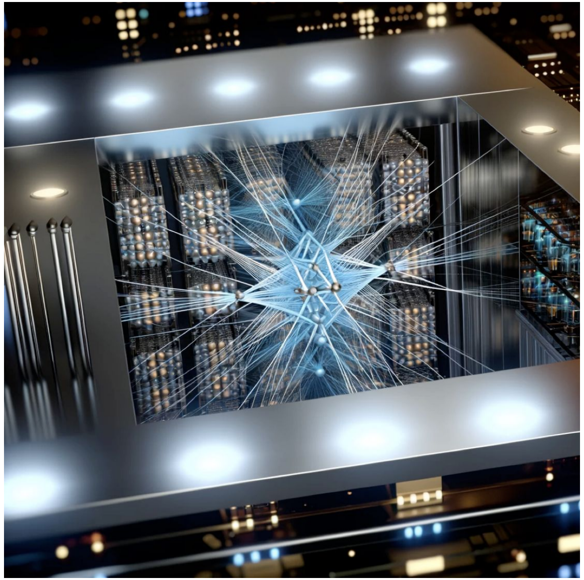

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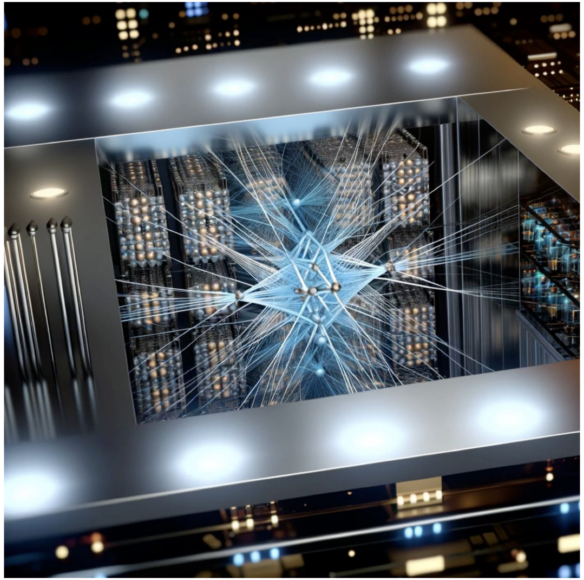

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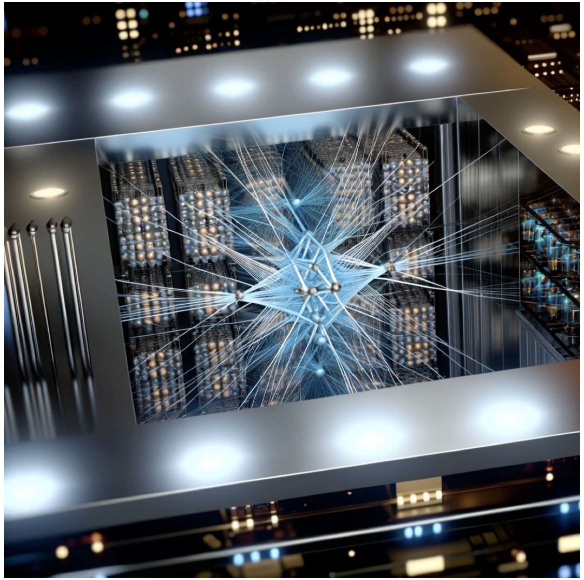
This is the central problem in the study of QLDPC codes.



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Logical Computation on QLDPC Codes through Surgery

This talk: QLDPC surgery is a method of logical computation that is fault-tolerant, addressable, universal, and low-overhead. Work done collectively in three papers.

1. Improved QLDPC Surgery: Logical Measurements and Bridging Codes.
Andrew Cross, **Zhiyang He**, Patrick Rall, Theodore Yoder. 2407.18393

2. Low-overhead fault-tolerant quantum computation by gauging logical operators.
Dominic Williamson, Theodore Yoder. 2410.02213

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Other related works: 2407.09423 (Cowtan), 2407.18490 (Xu et al.), 2408.01339 (Zhang, Li), 2410.02753 (Ide et al.).

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Logical Measurement and Lattice Surgery

Pauli-based computation: Pauli measurements on logical qubits + magic states = universal computation.

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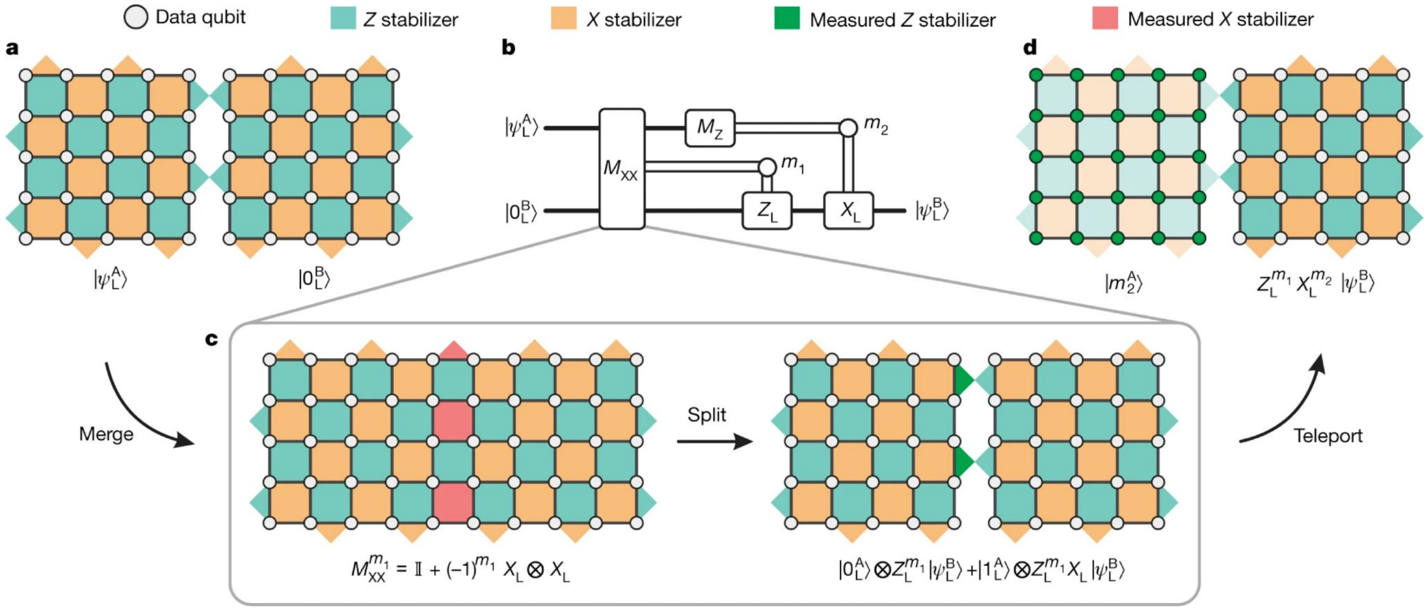

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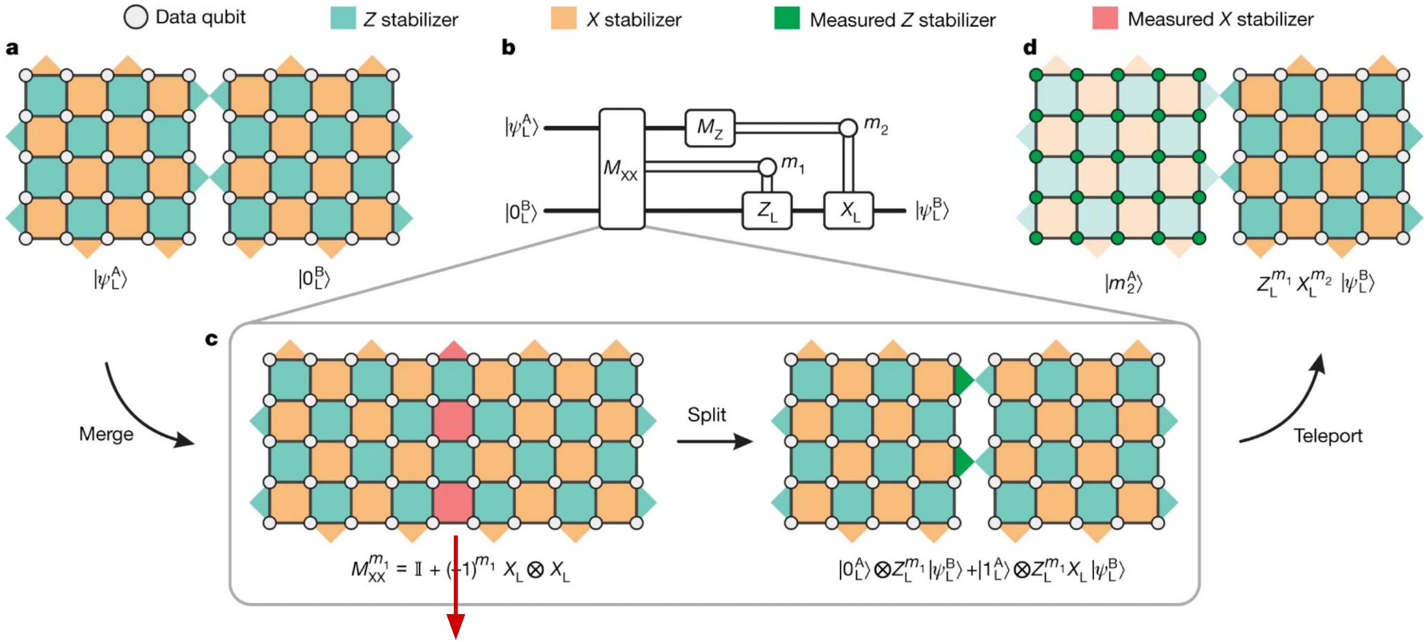

Product of red X-checks = $X_L \otimes X_L$ – obtain logical measurement result by measuring new stabilizers.

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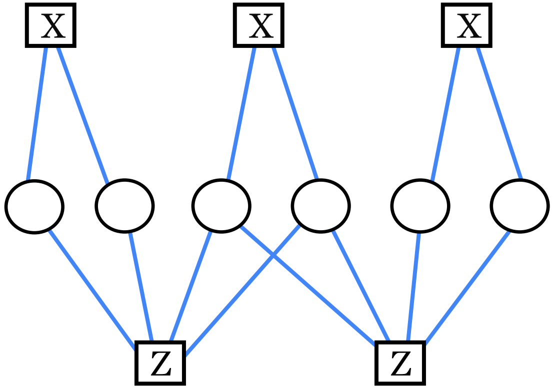
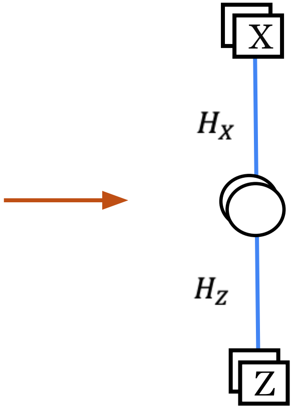
Quantum LDPC Surgery

Quantum LDPC Surgery

Tanner graph of code



Shorthand form



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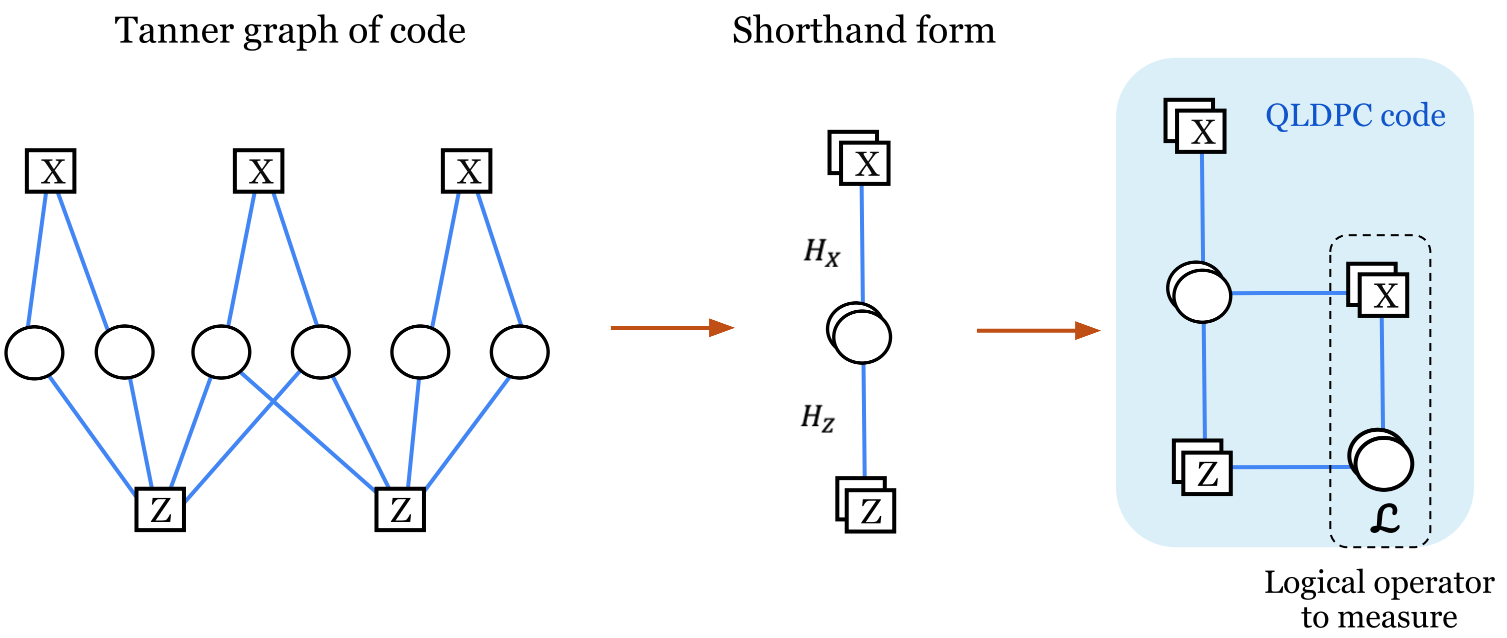

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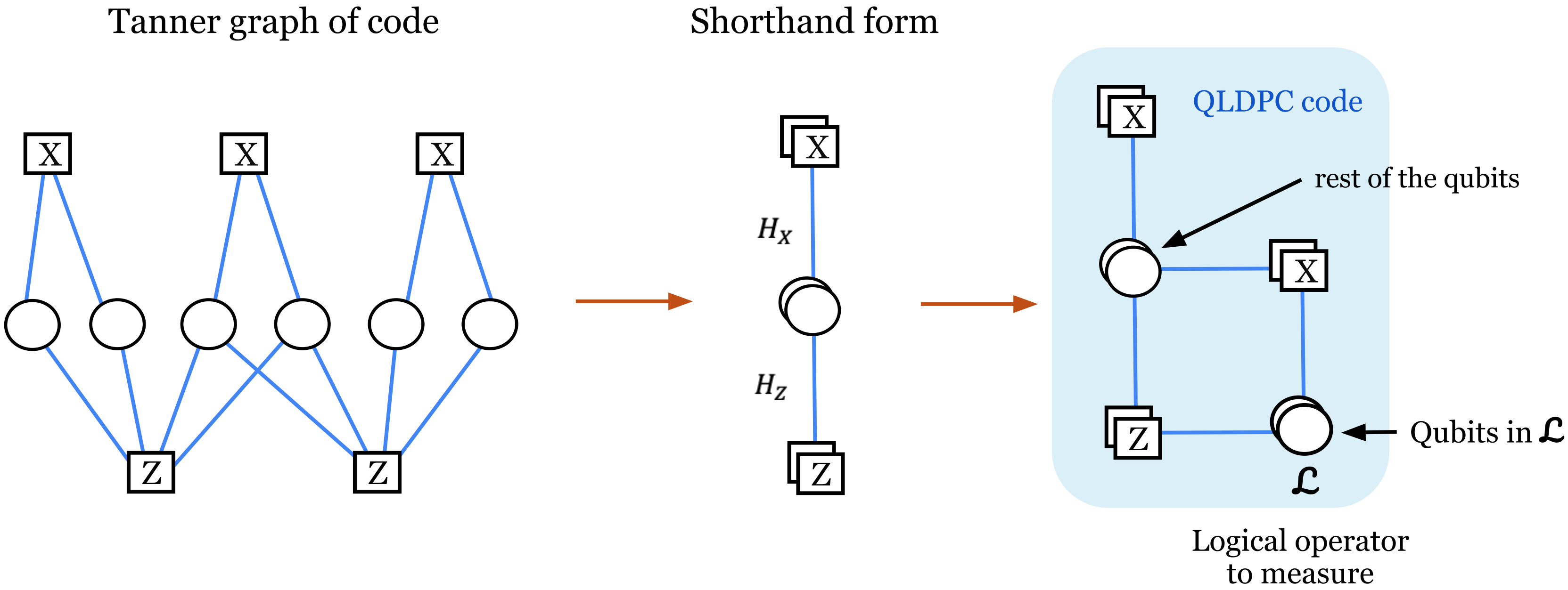

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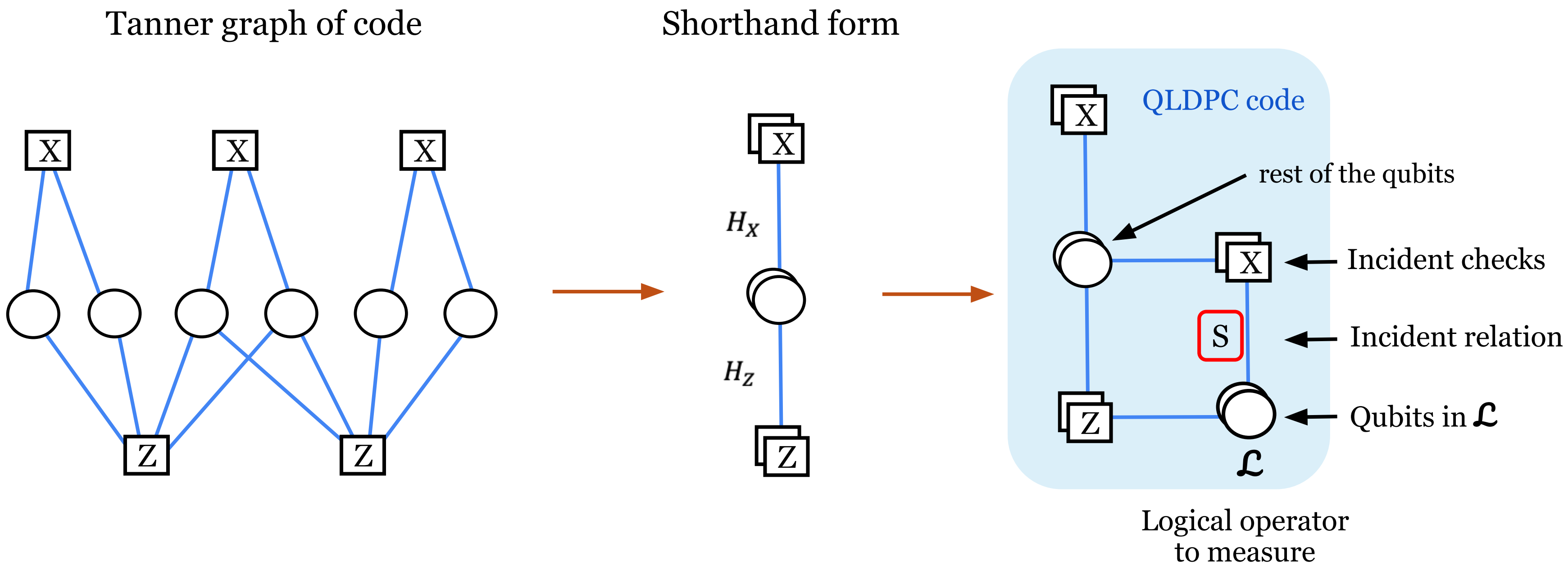
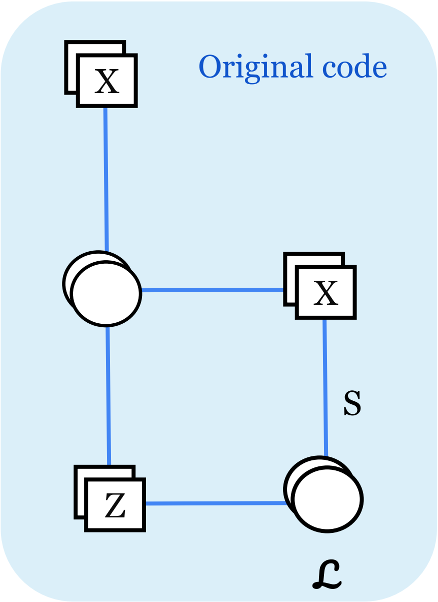


CKBB Surgery Method [2110.10794]



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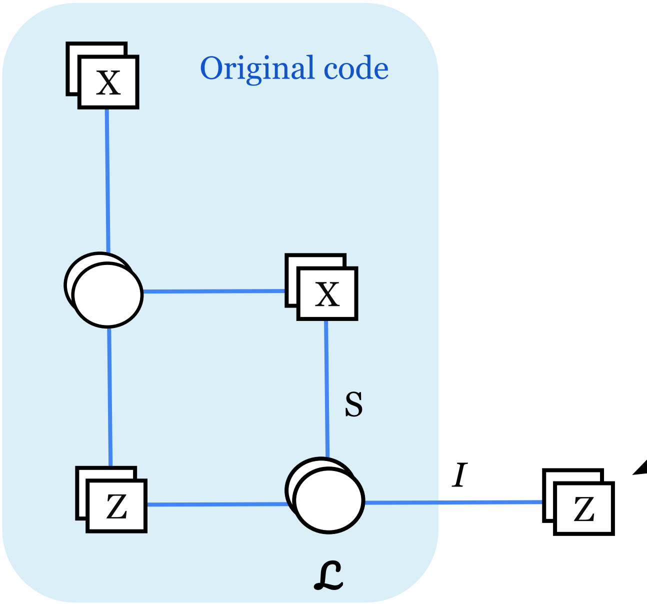

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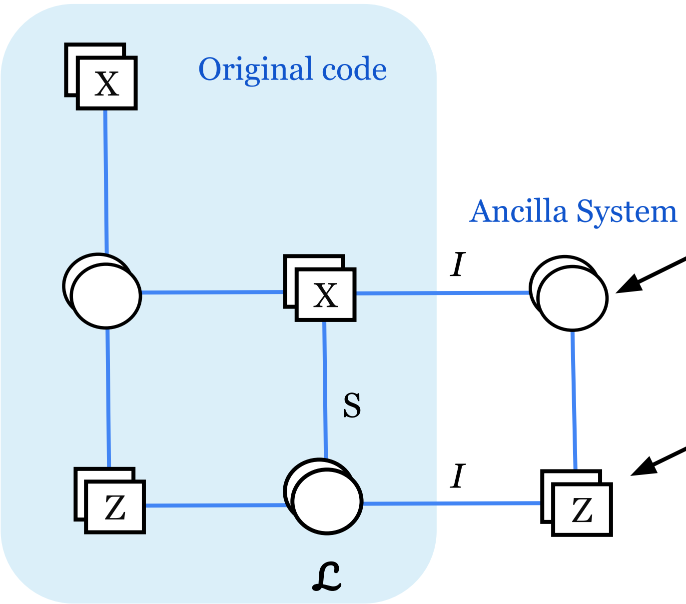

Add one ancilla qubit for each X check.

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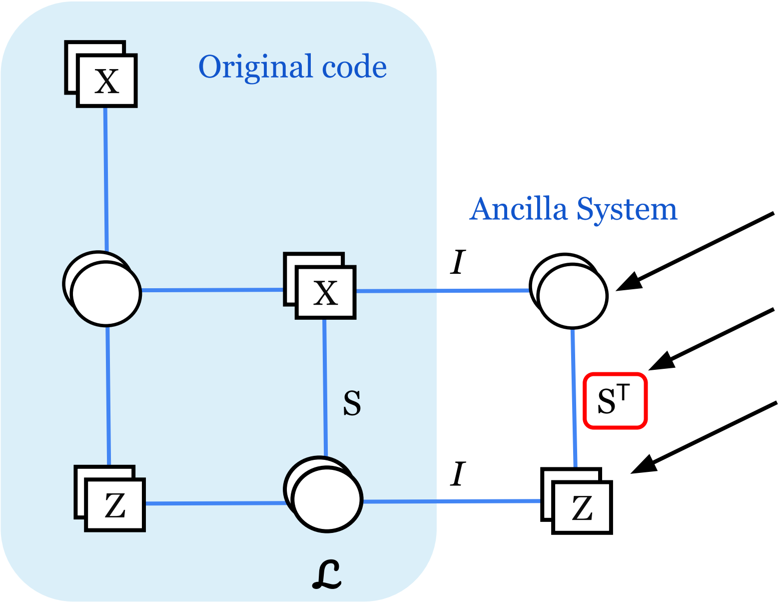

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Same incidence relation as in $\mathcal{L}$.

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Low-overhead fault-tolerant quantum computing using long-range connectivity; [Cohen, Kim, Bartlett, Brown 2021].

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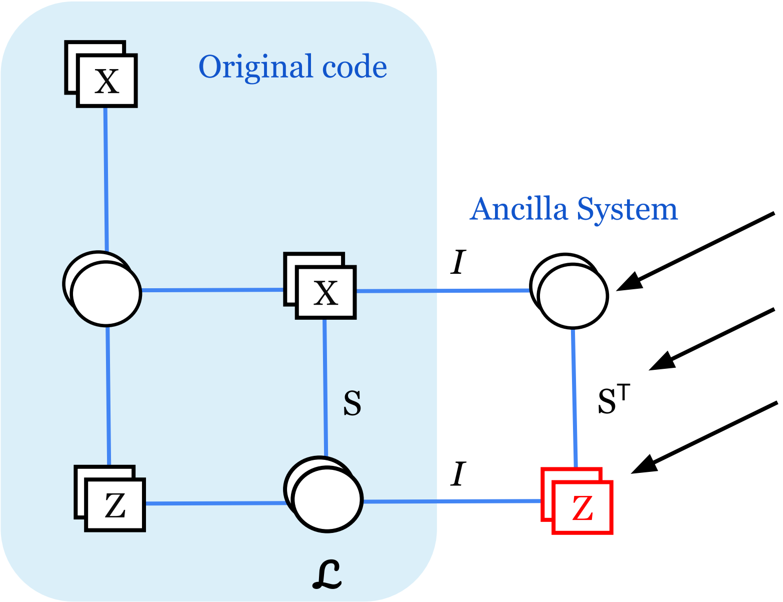

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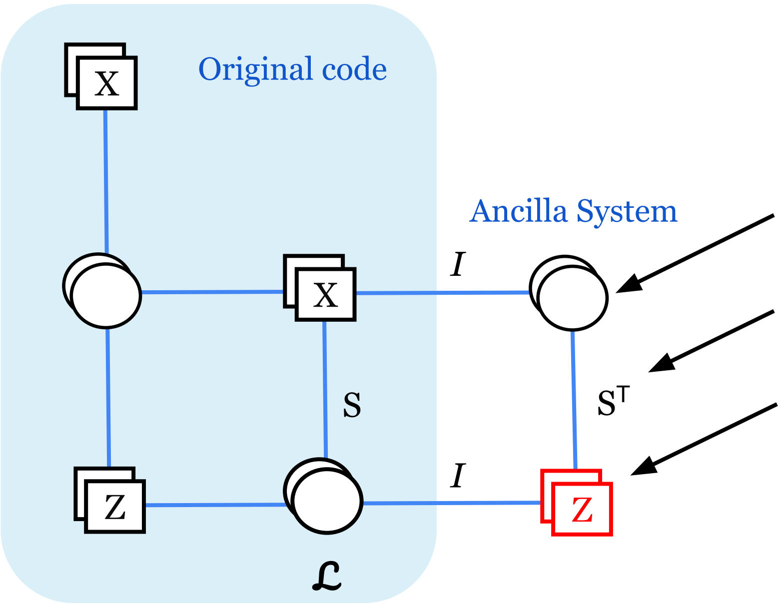

Are we done?

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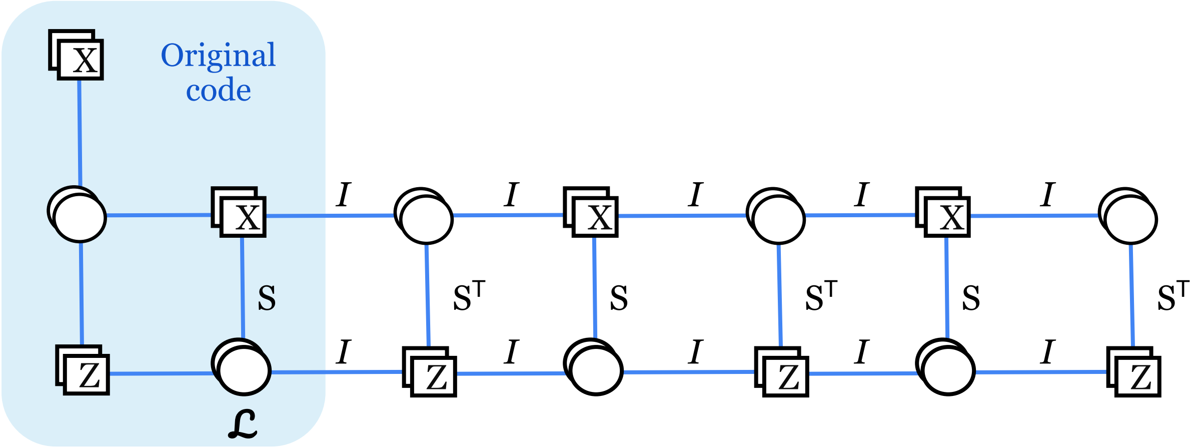

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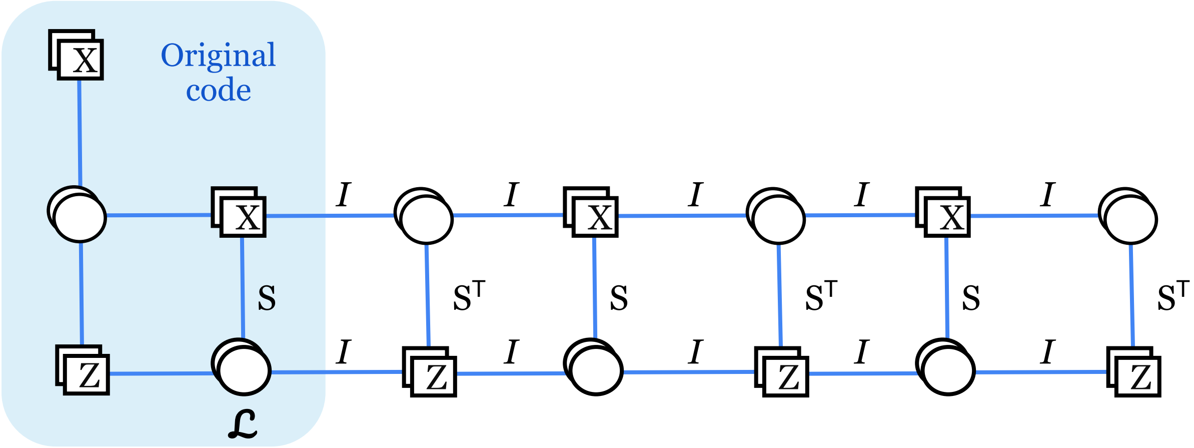

Advantage: applicable to any QLDPC code,

Issue: space overhead ~ $O(d^2)$, similar to surface code!

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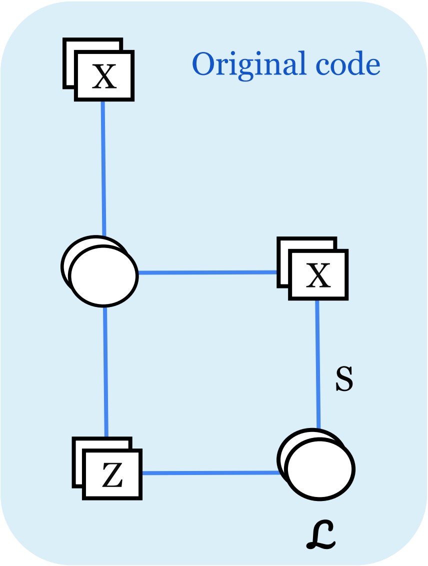

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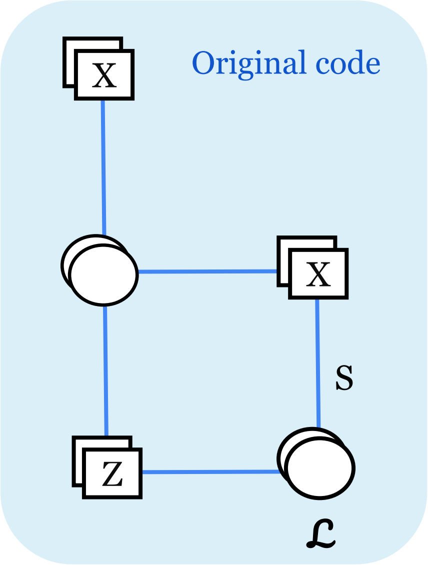

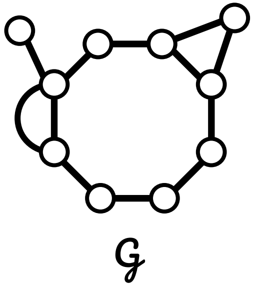

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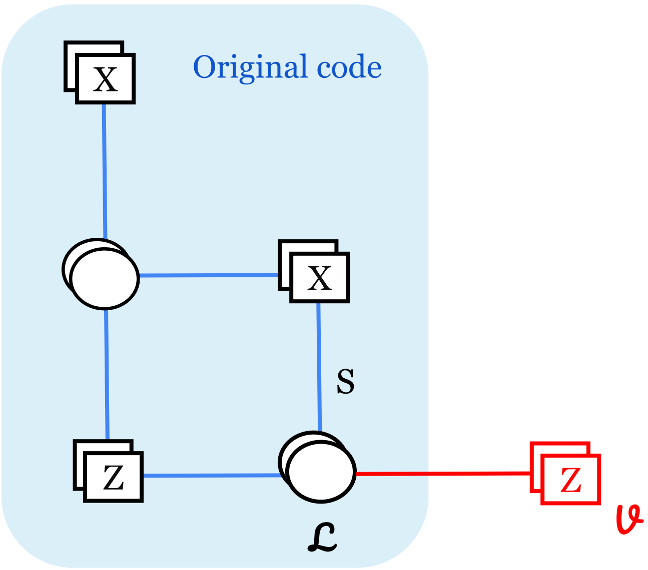

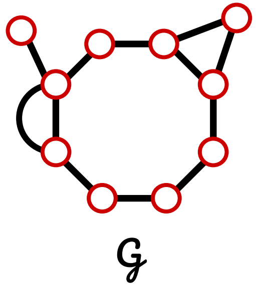

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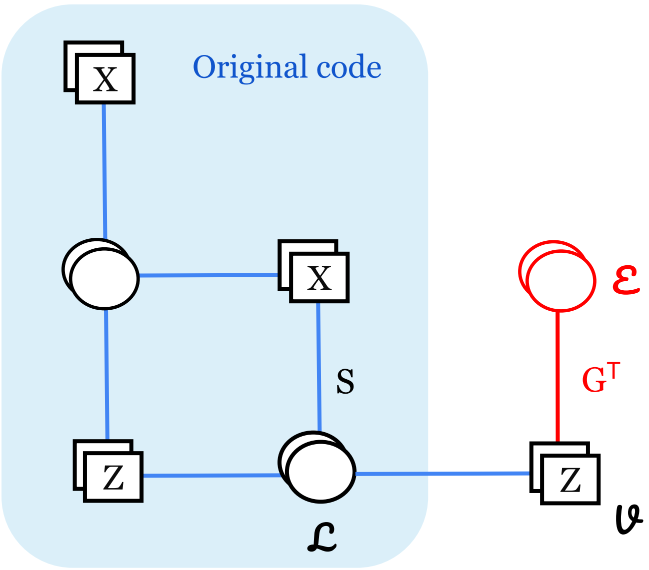

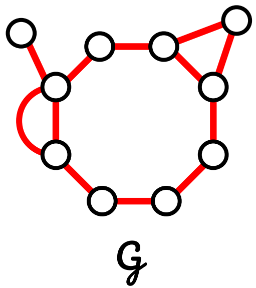

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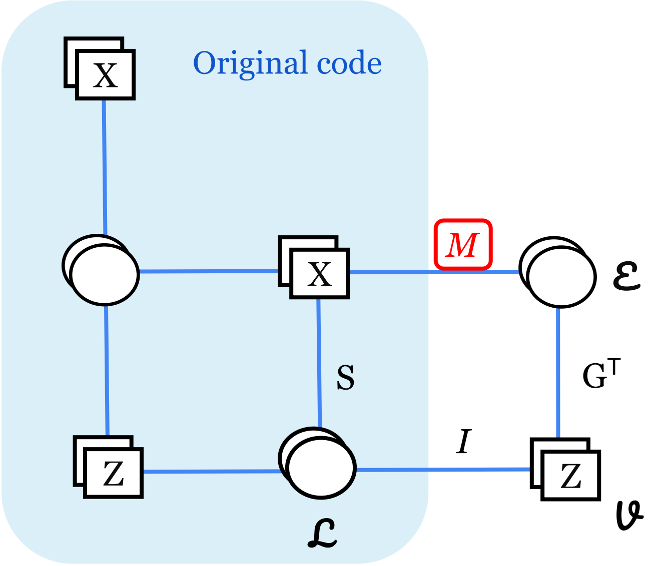

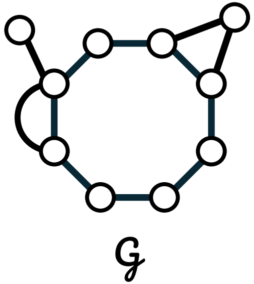

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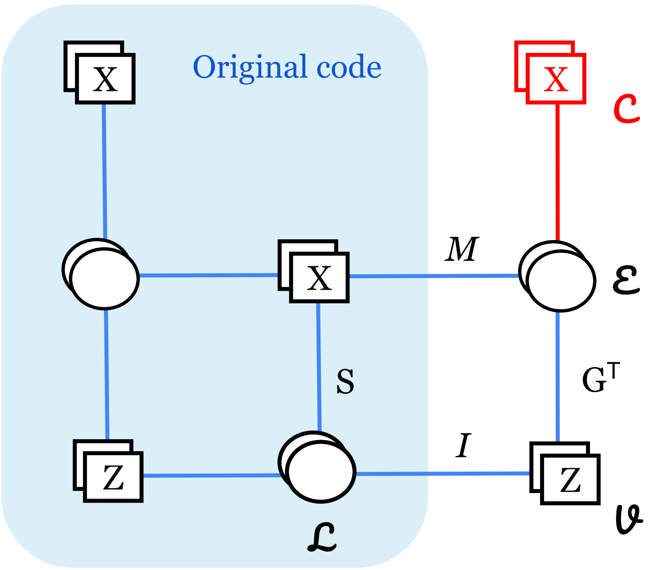

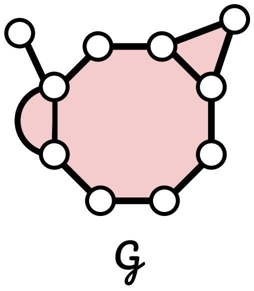

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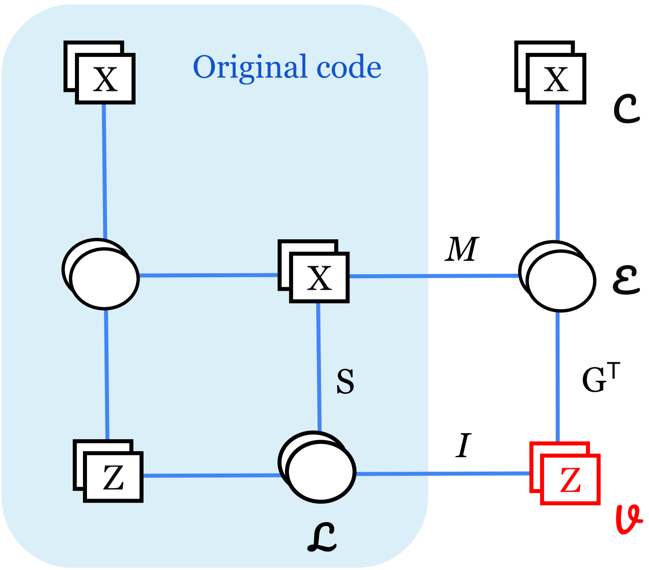

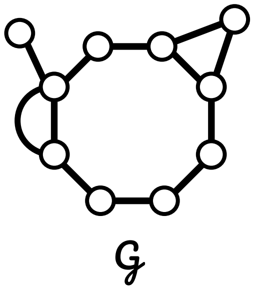

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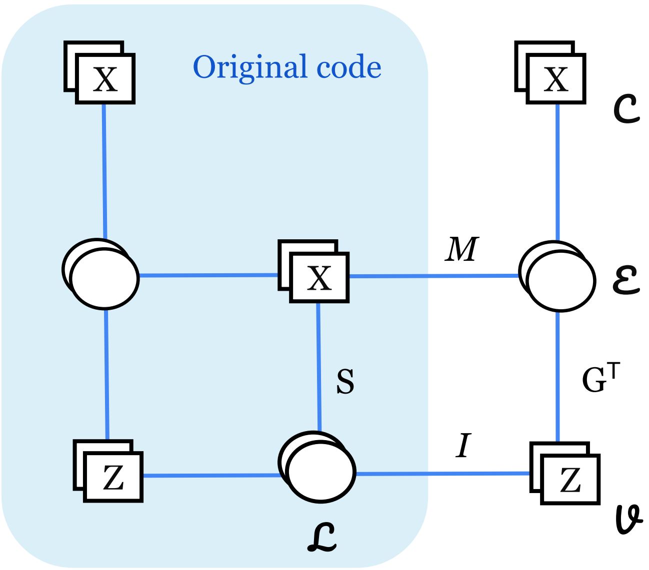

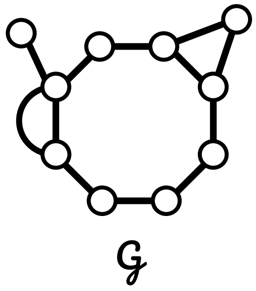

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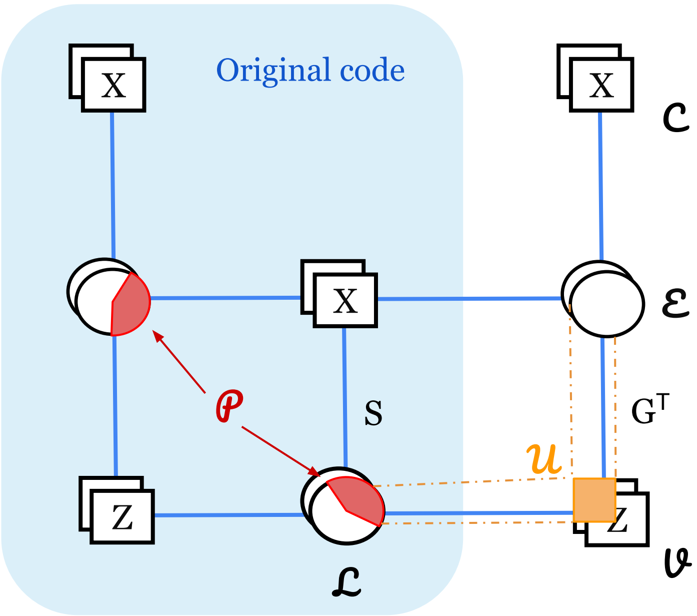
OK… what about code distance?

Expansion brings fault-tolerance



Let $\mathcal{P}$ be another Z operator, $\mathcal{U}$ be a set of ancilla checks,

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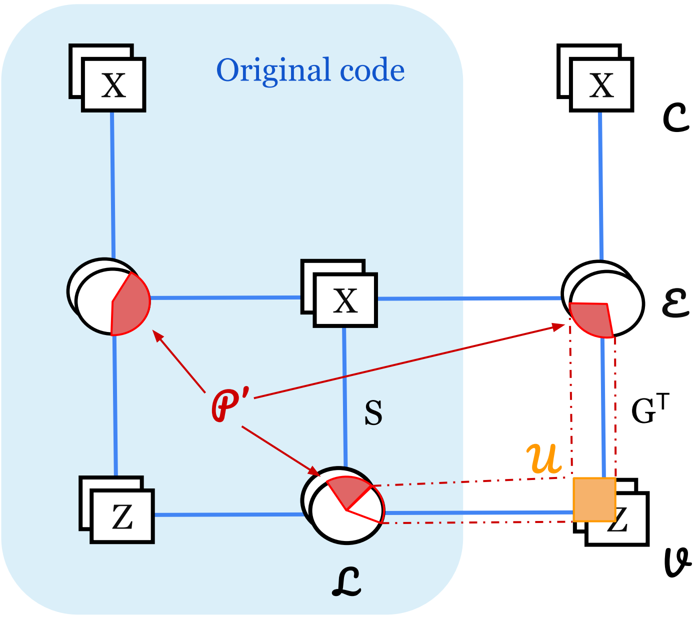

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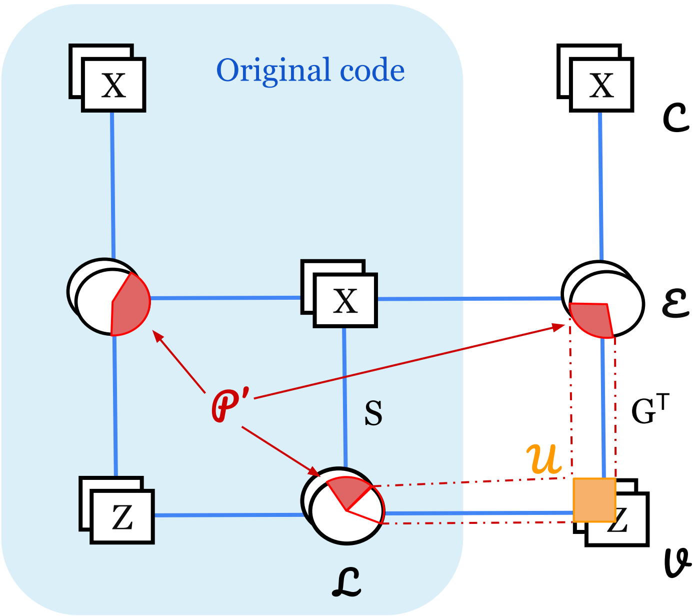

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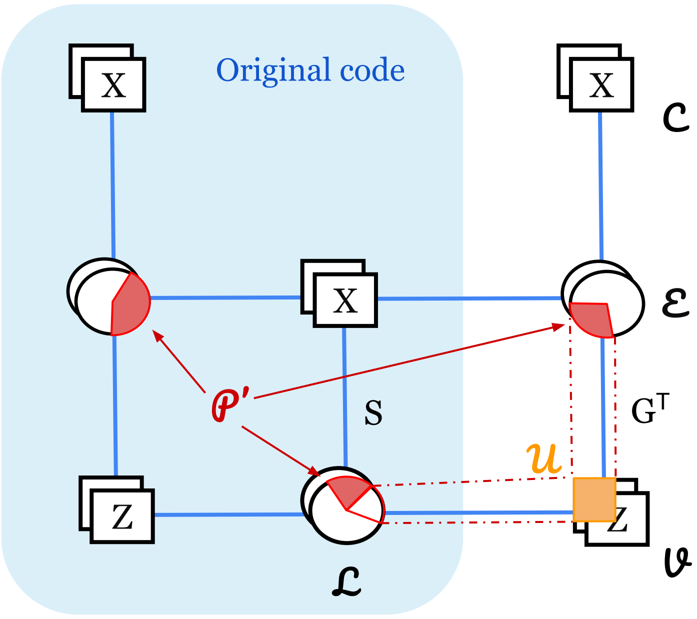

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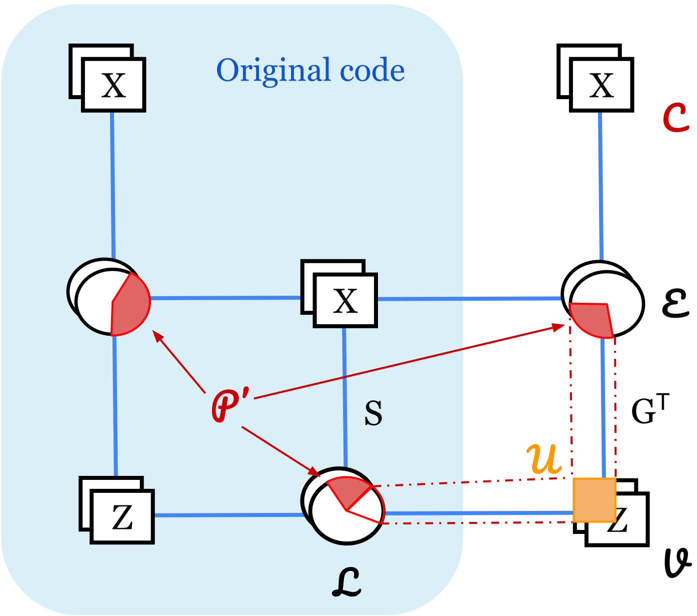

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Other ways to reduce distance?

Not if we measure the cycles $\mathcal{C}$.

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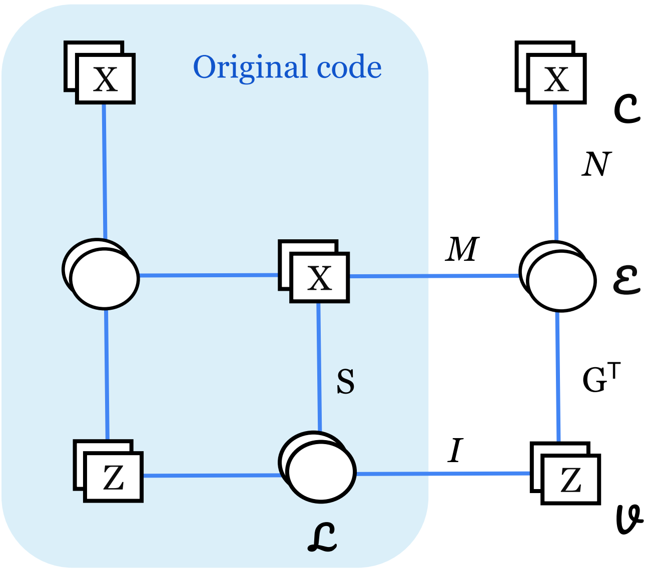
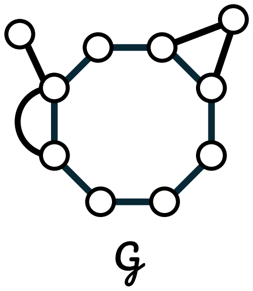
- Case Study: [[144,12,12]] Bivariate bicyclic code

Graph Desiderata





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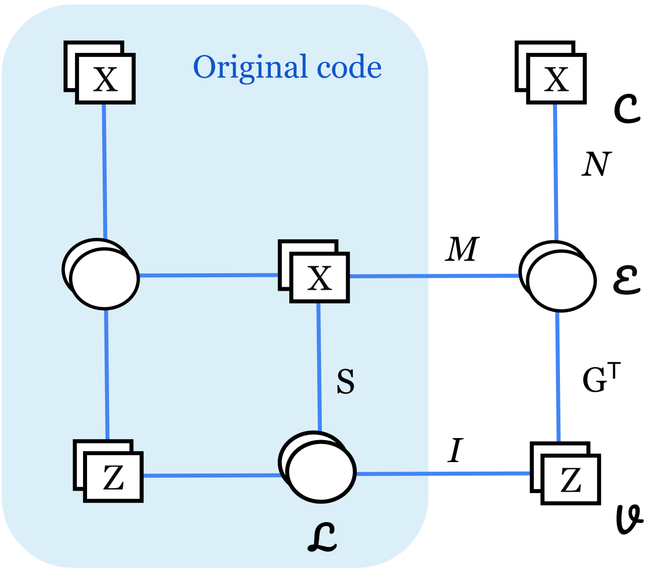

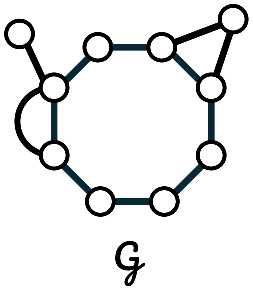

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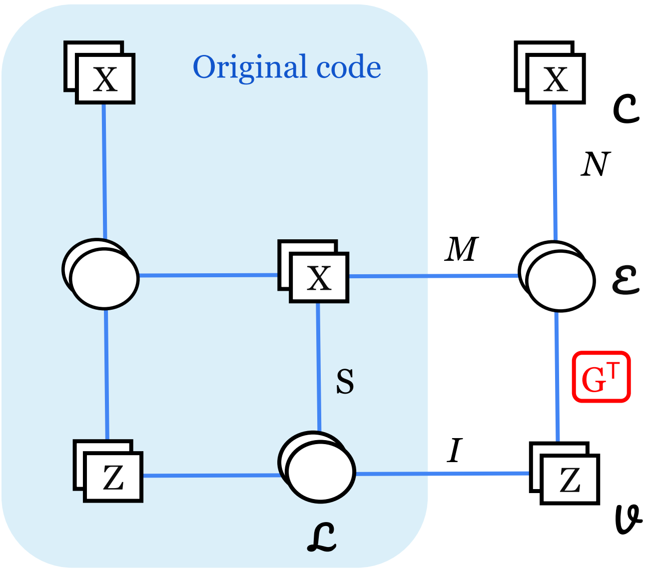

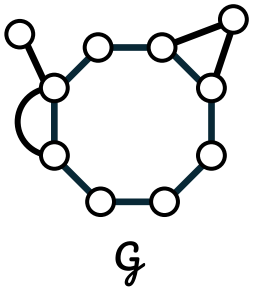

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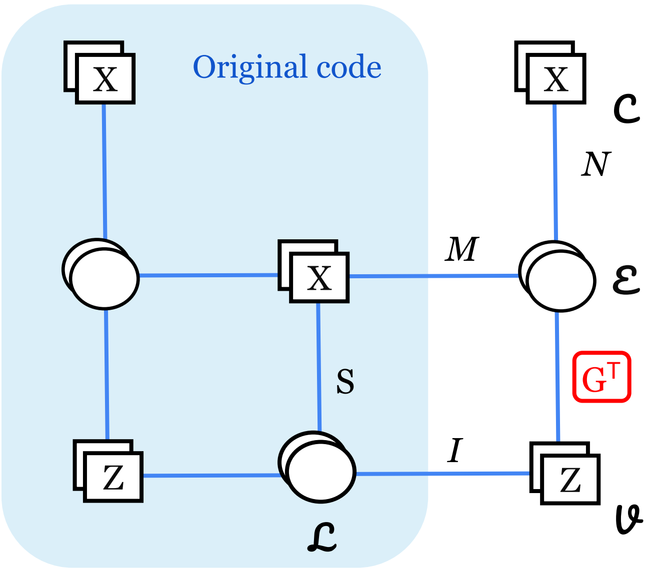

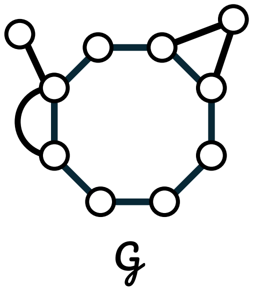

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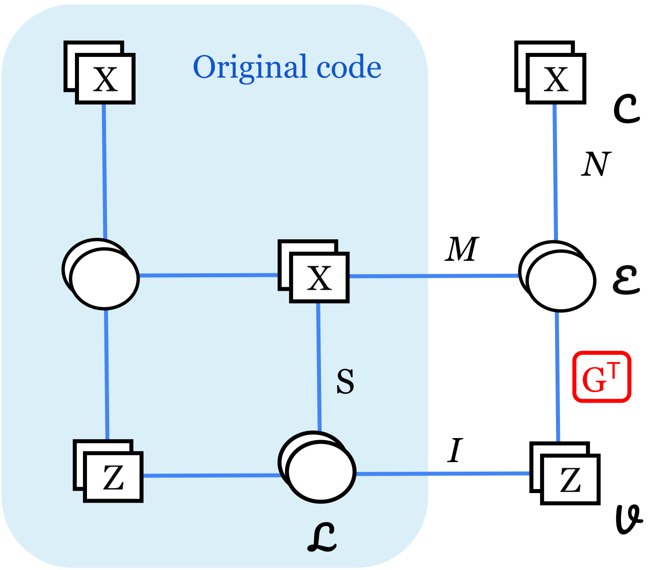

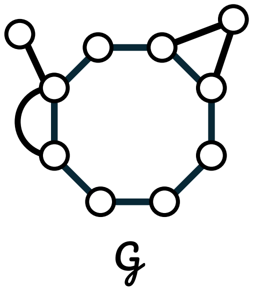

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Ingredient #1: choose a randomly constructed constant-degree expander graph.

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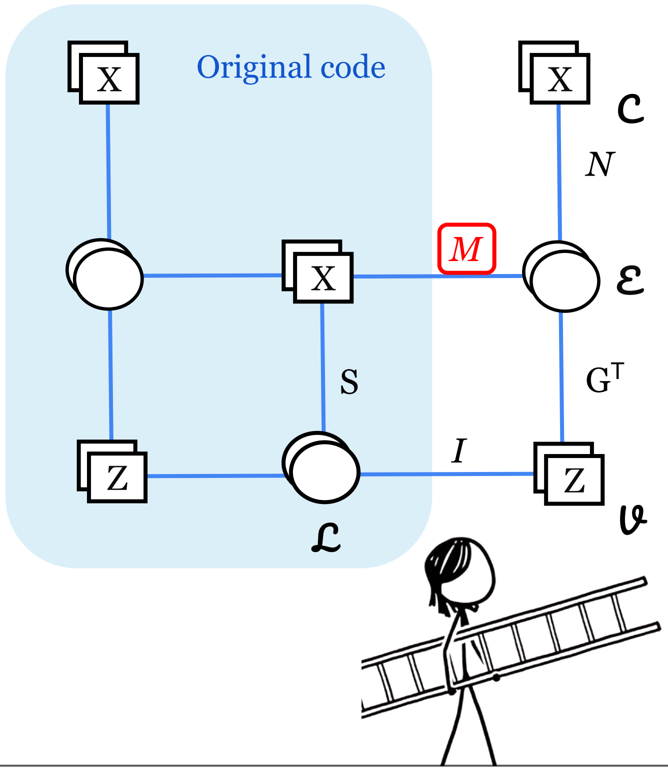

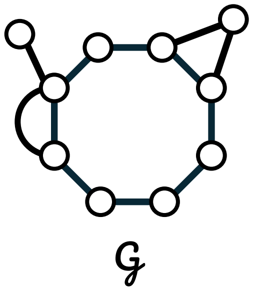

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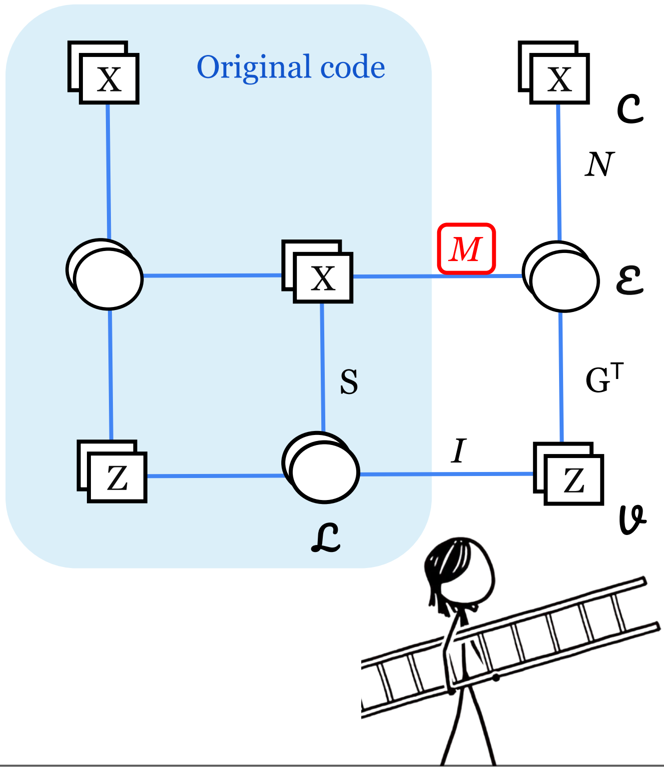

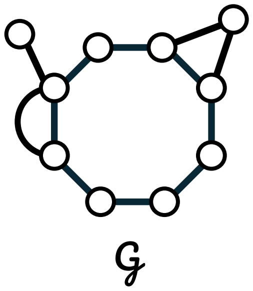

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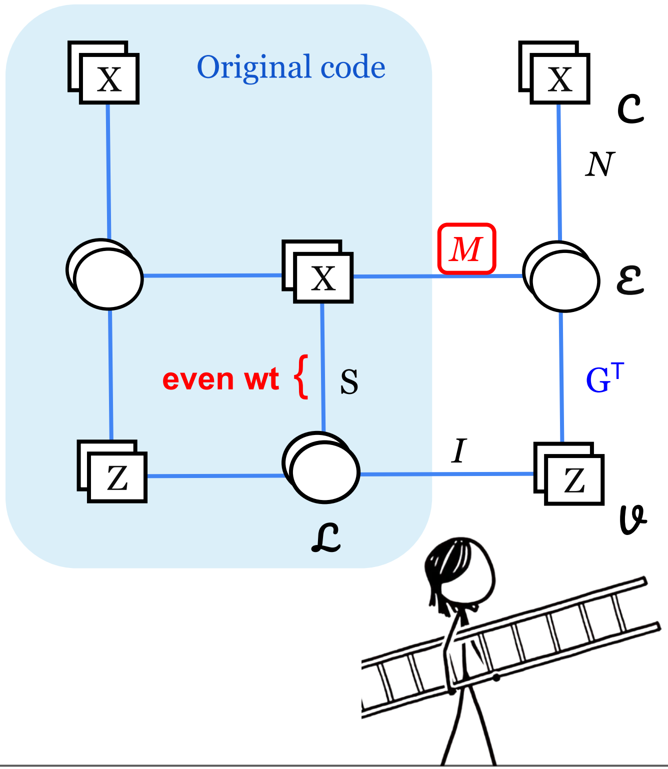

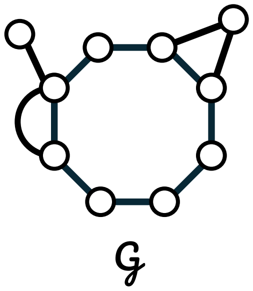

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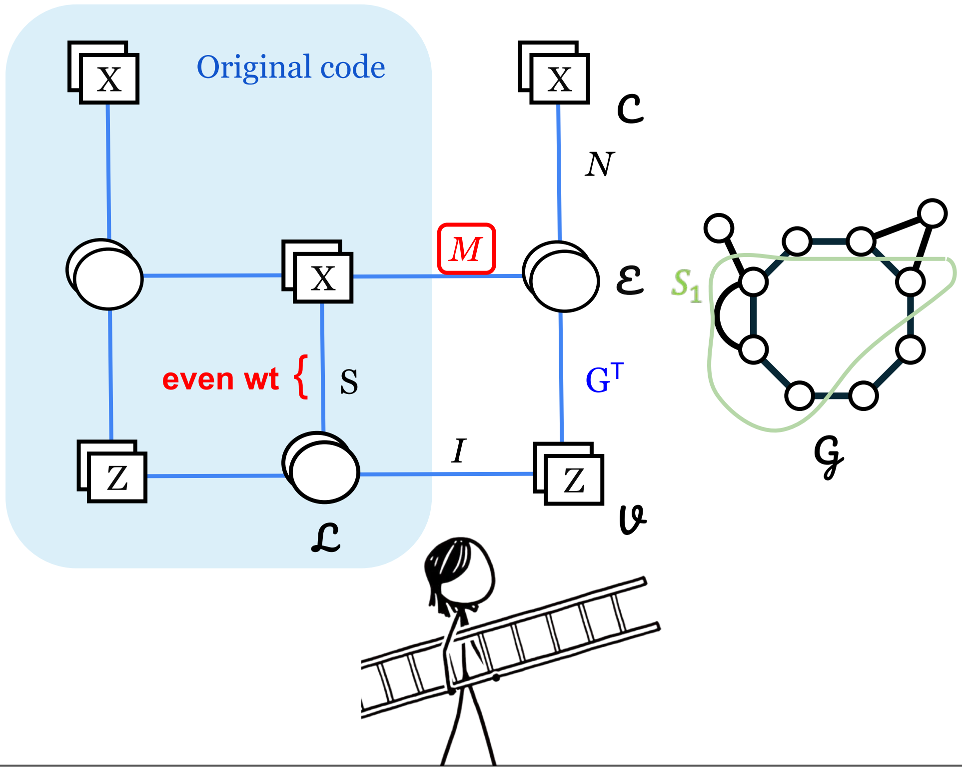

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⟺ original X stabilizers anti-commute with an even number of new vertex Z stabilizers $\mathcal{V}$.

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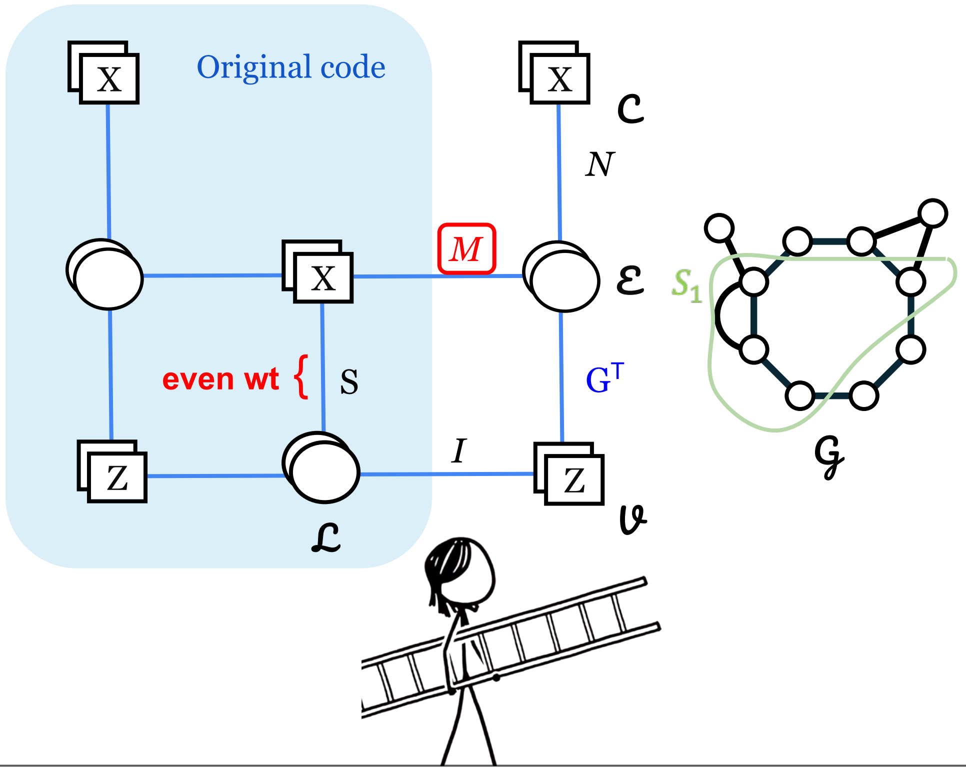

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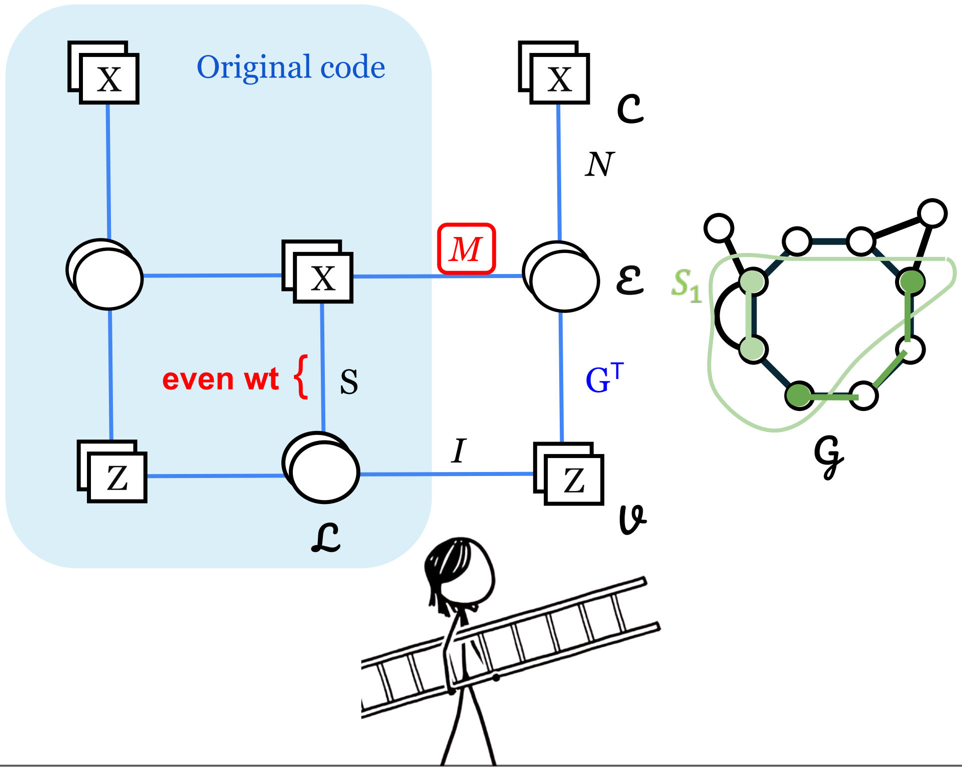

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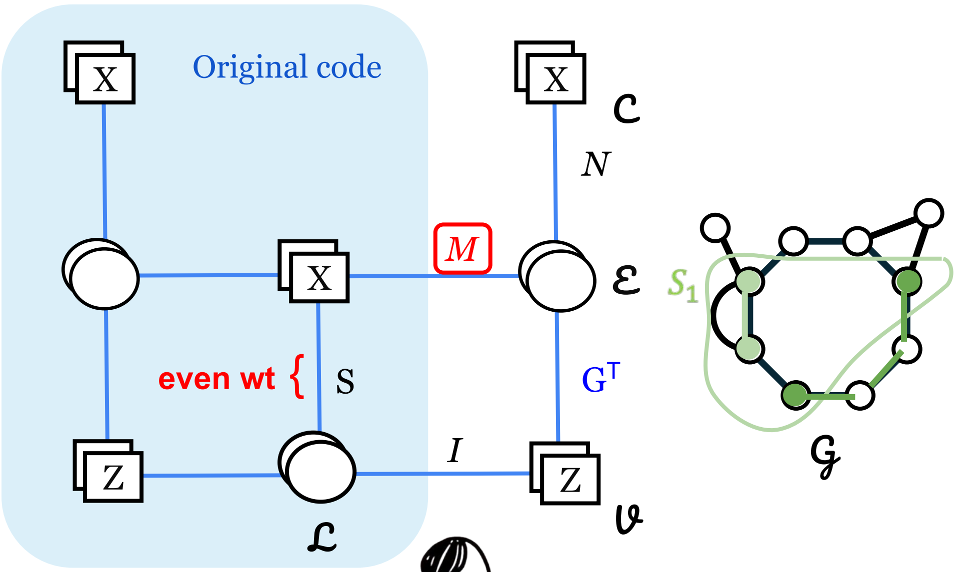

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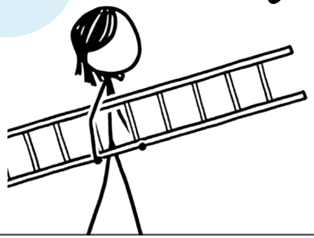
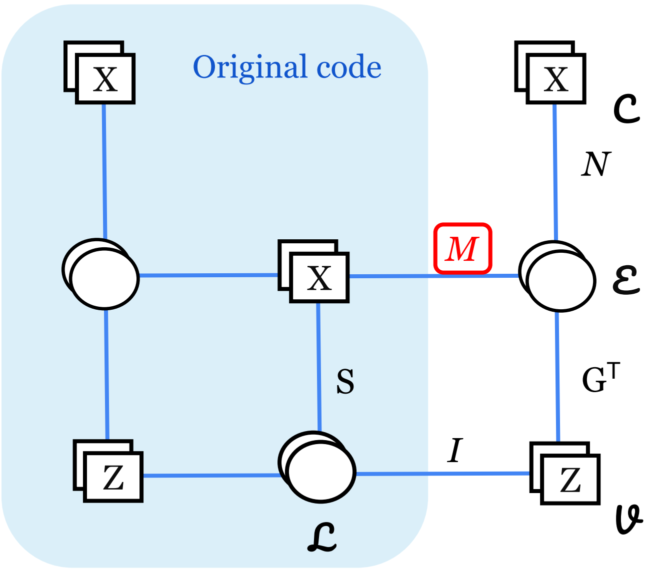
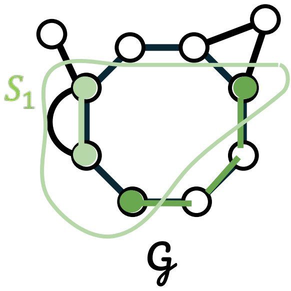


Ingredient #2: come up with a (short) perfect matching on $\mathcal{G}$

Graph Desiderata





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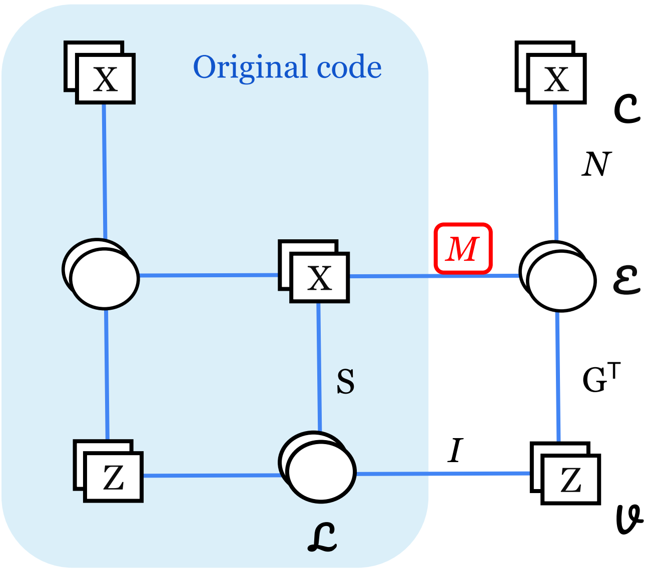

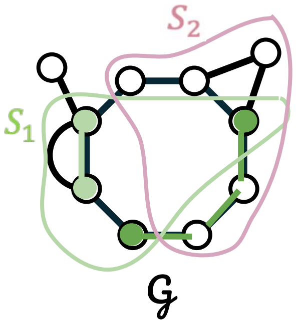

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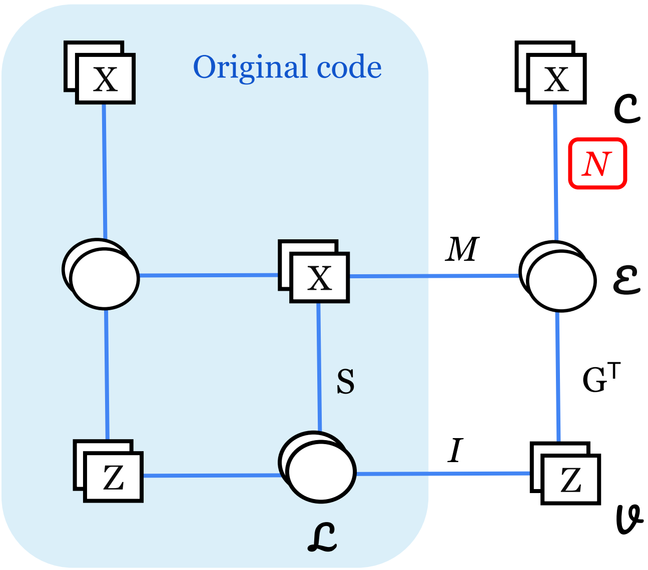

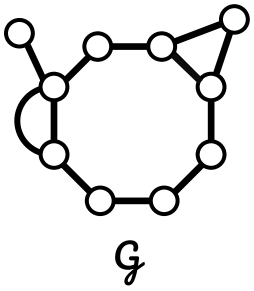

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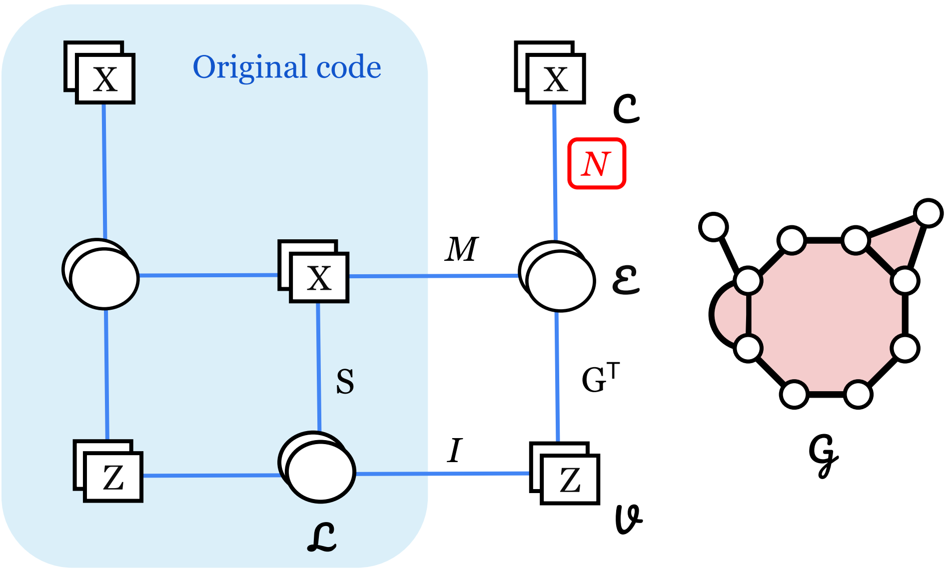

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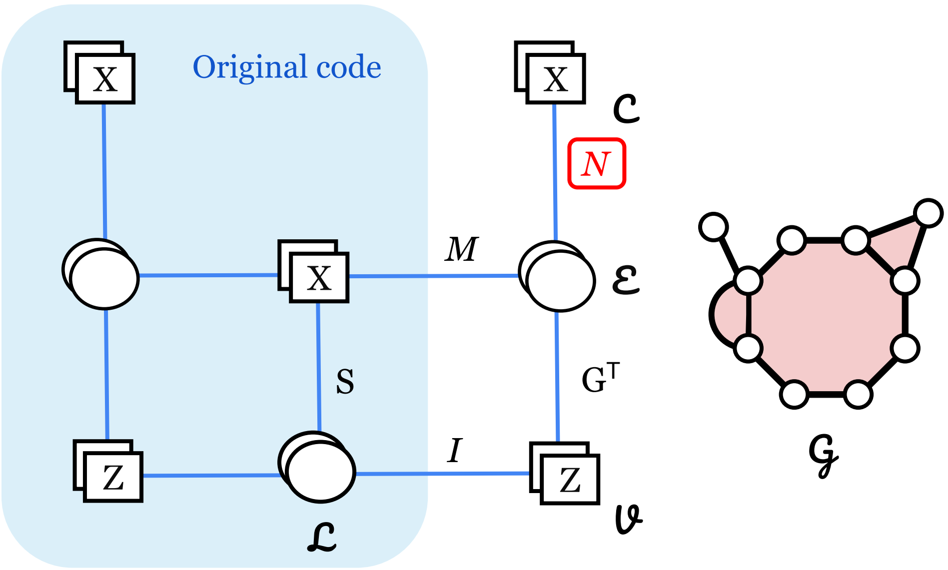

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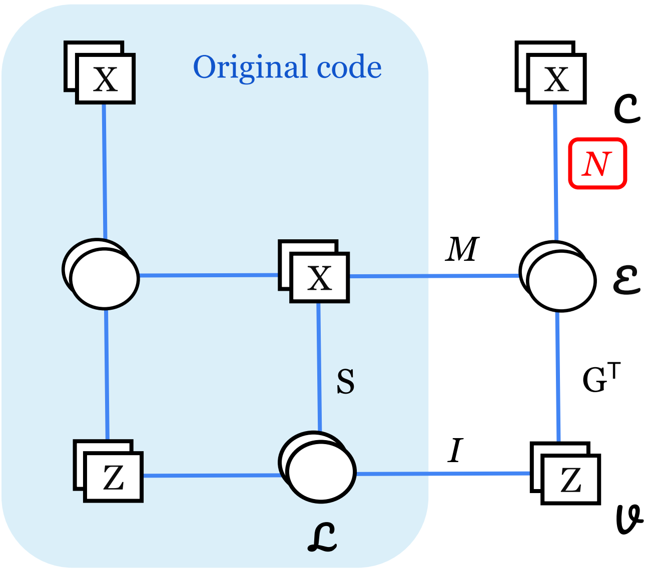

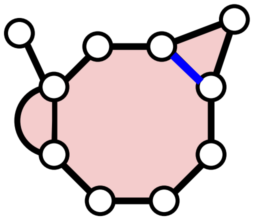

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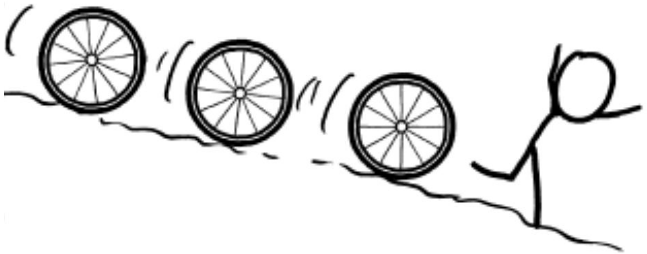

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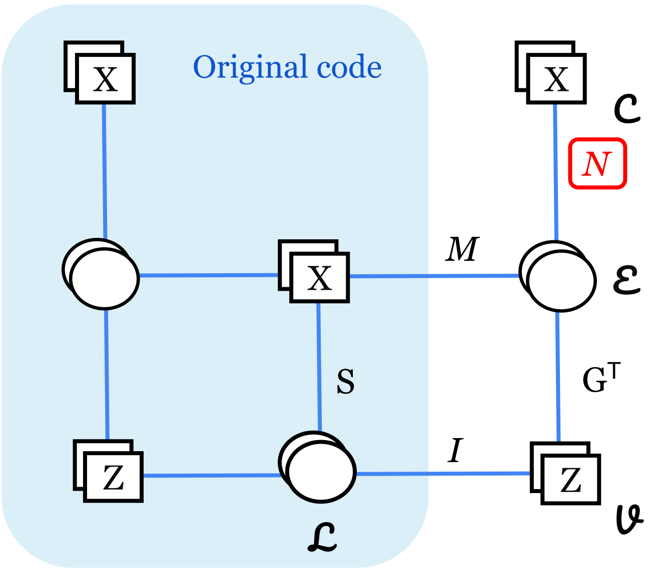

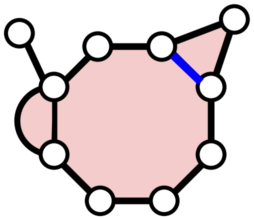

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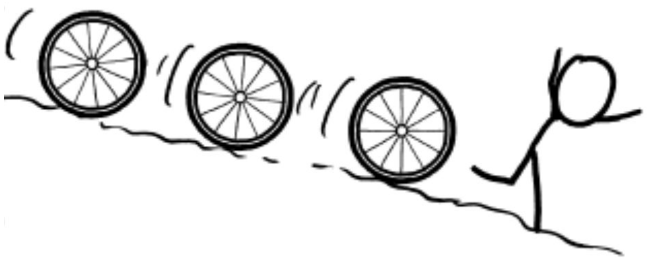

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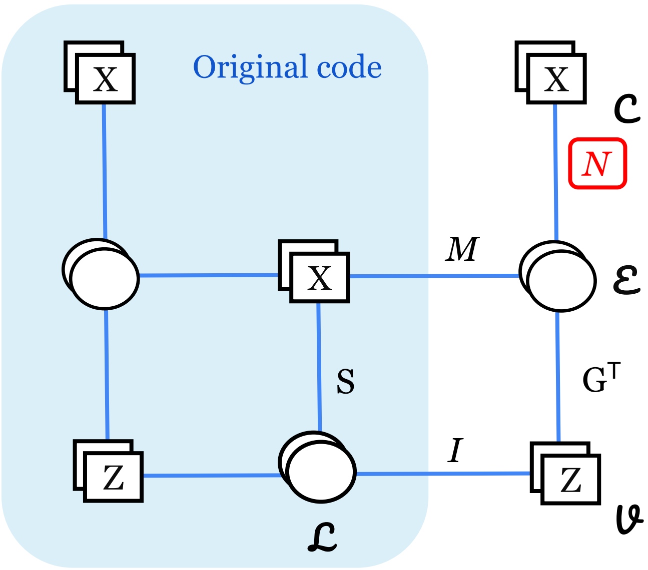

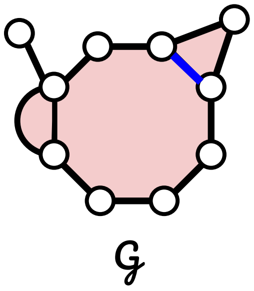

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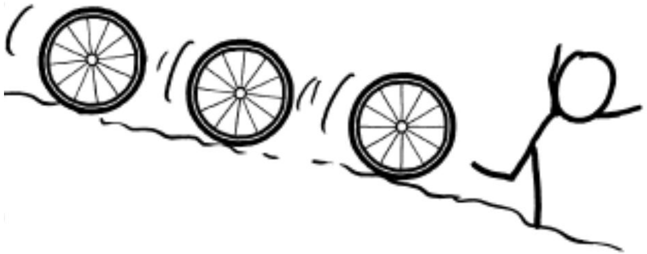

- Each edge isn't in too many cycles
- Each cycle* isn't too long.

*element of the cycle basis

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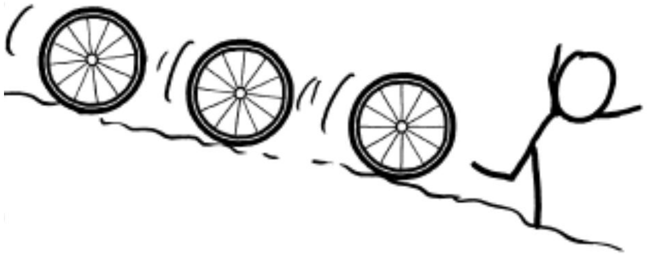
Ensuring the cycle basis of graph is sparse :

- Each edge appears only in O(1) cycle basis elements

Ingredient #3: a notion of "decongesting" cycles in a graph



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A cycle basis to begin with [Freedman Hastings 2020]

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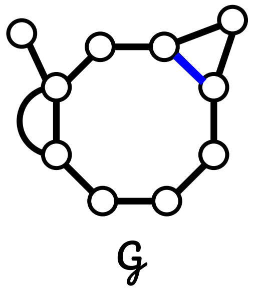

Cycles = X-checks

Edges = qubits

A cycle basis to begin with [Freedman Hastings 2020]

Input: graph $\mathcal{G}$ with O(1) vertex degree

Output: a cycle basis s.t.

each cycle overlaps with at most **$O(\log^3(|V|))$** cycles.

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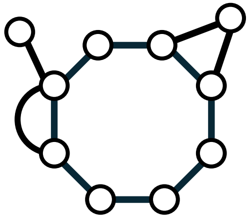

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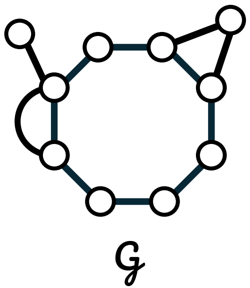

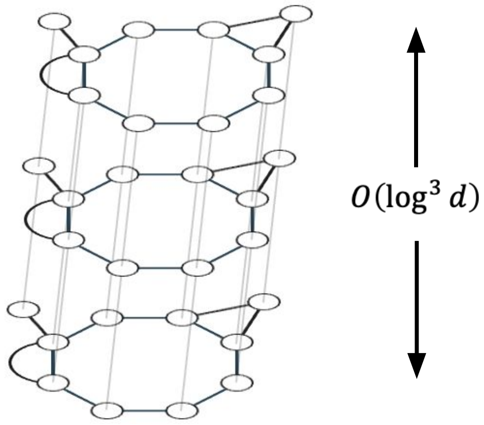

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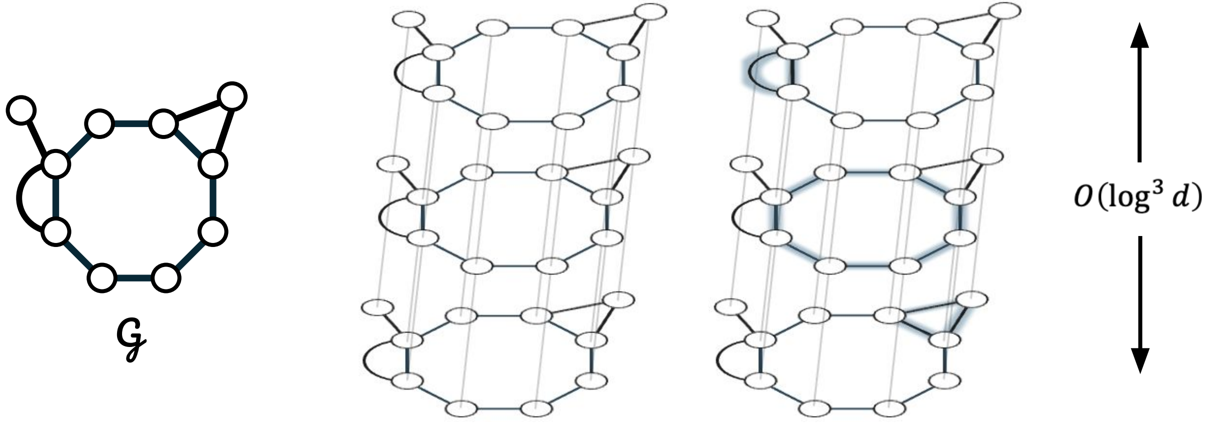

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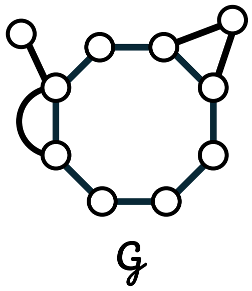
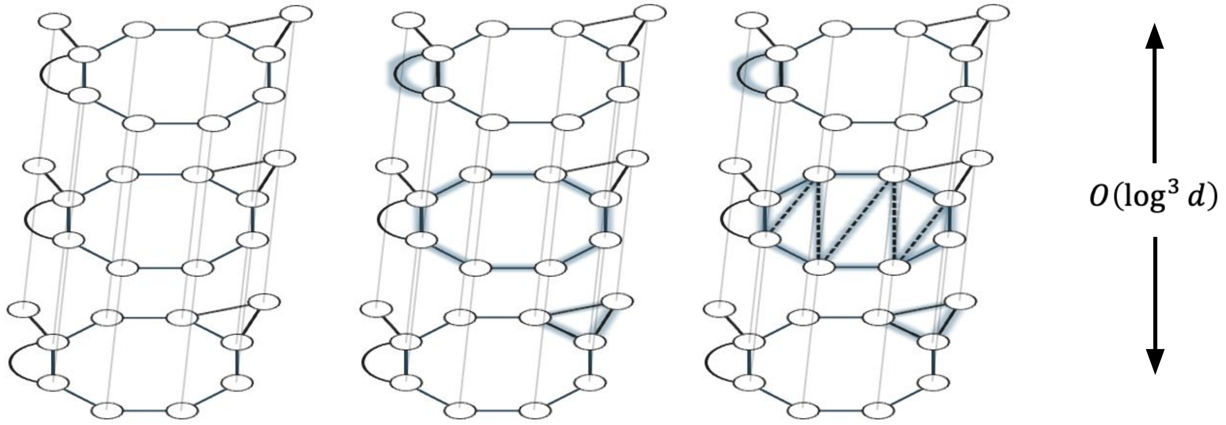
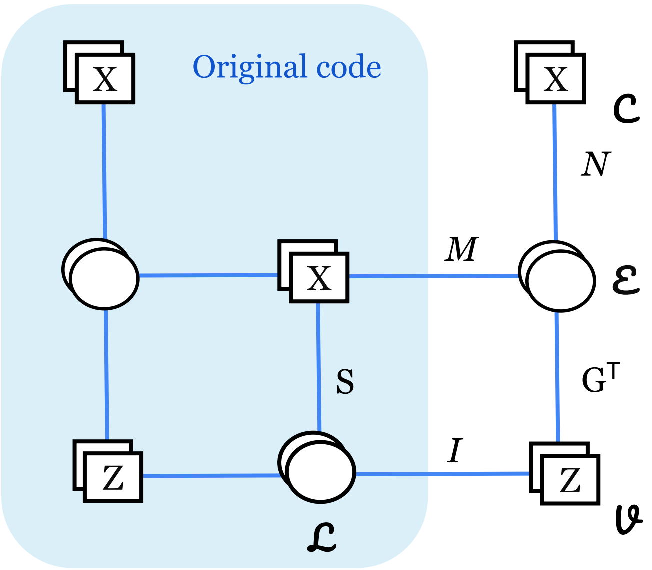


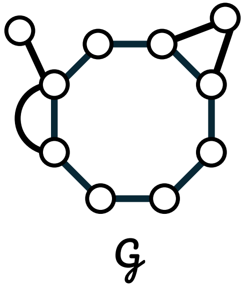


Also "cellulate" long cycles into smaller cycles

- Each cycle basis element has O(1) edges

Graph Desiderata





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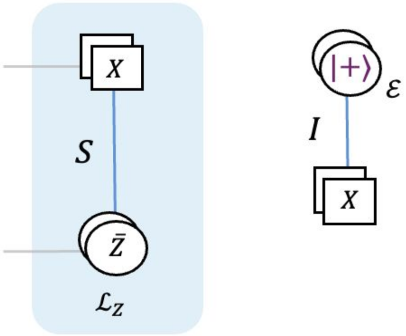
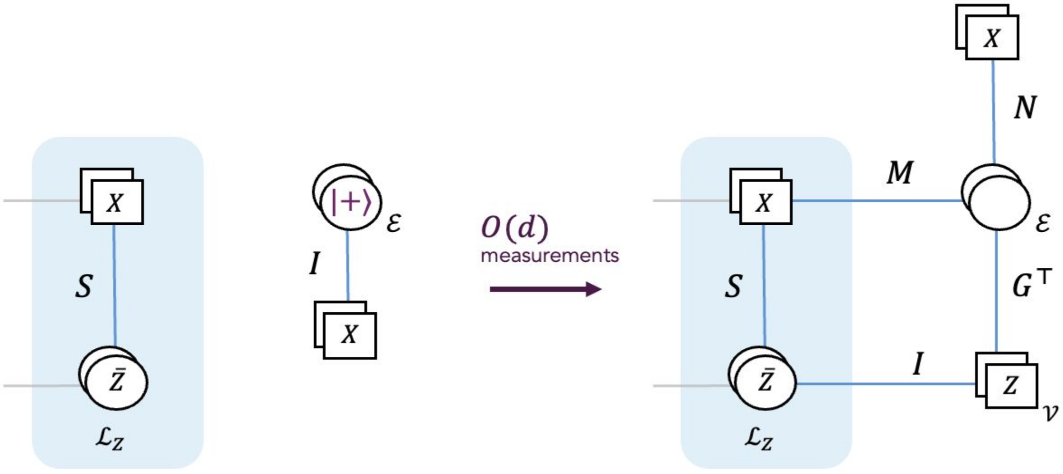
3. Short perfect matchings on $\mathcal{G}$ (For original X checks) ✓ & each edge is in O(1) matchings. ✓
4. $\mathcal{G}$ has a sparse cycle basis. ✓

Overall protocol for auxiliary graph qLDPC surgery



(i) initialize

Overall protocol for auxiliary graph qLDPC surgery



(i) initialize

(ii) merge step

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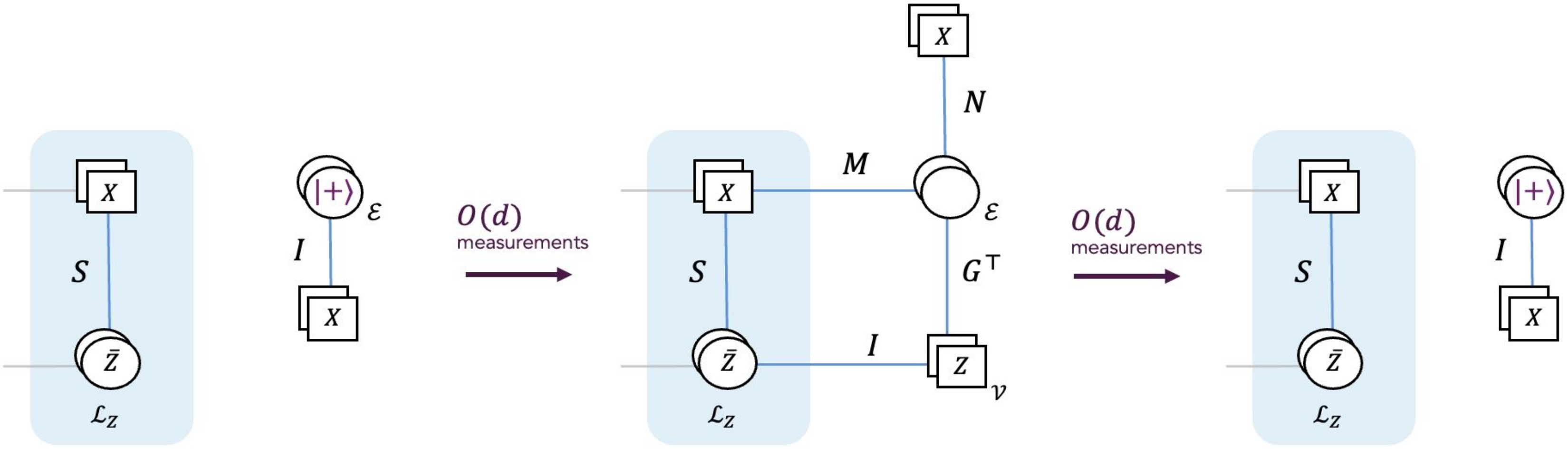
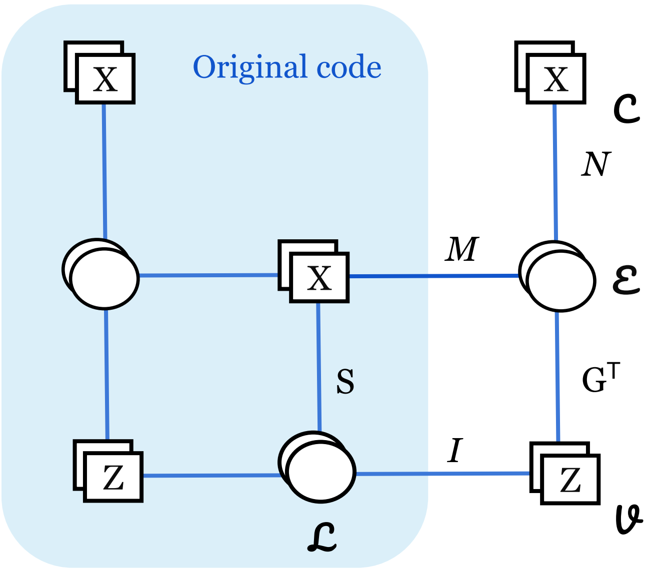


(i) initialize

(ii) merge step

(iii) split step

In summary: auxiliary graph qLDPC surgery



✔ **Applicable to any quantum LDPC code!**

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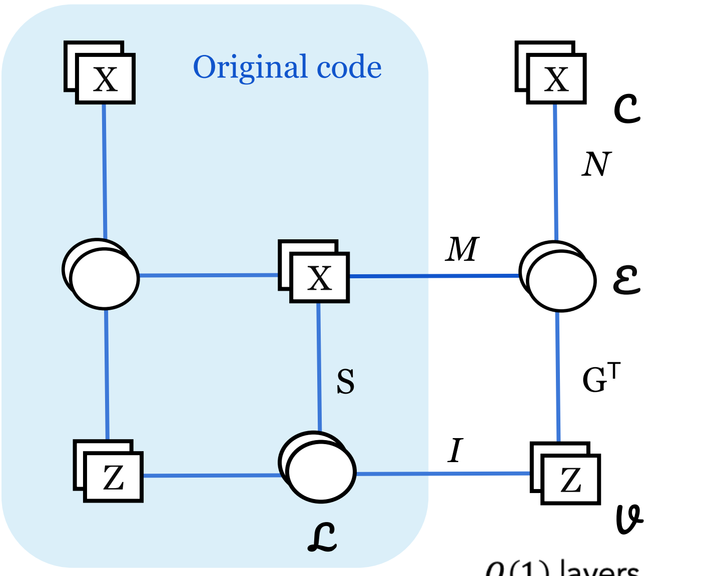

$O(1)$ layers
(best-case)
to
$O(\log^3 d)$ layers
(worst-case)

✔ **Applicable to any quantum LDPC code!**

Qubit overhead of scheme: **$O(d \log^3 (d))$**

~ **O(d) in distance d, upto polylog**

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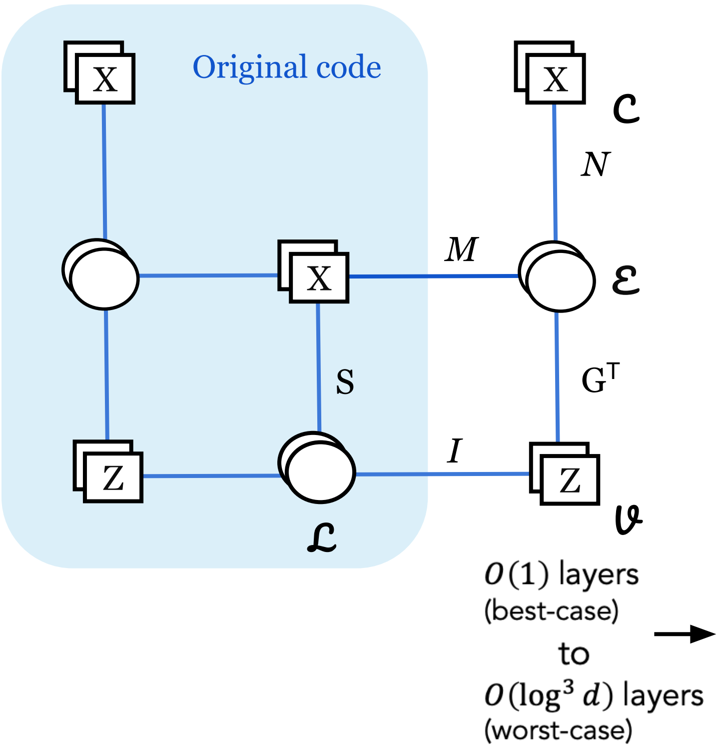

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⇒ Significant improvement in overhead from previous scheme for arbitrary quantum LDPC codes, $O(d^2)$

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Joint measurements of **products of logical Pauli operators, say $X_1Z_2Z_3$**, etc.. (without simultaneously measuring individual Paulis)

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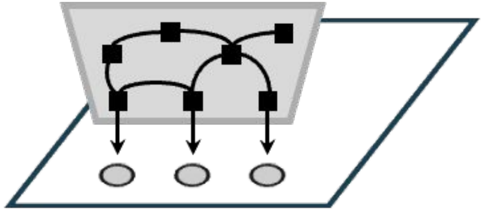
Construct a large **auxiliary graph** on the **entire logical**.



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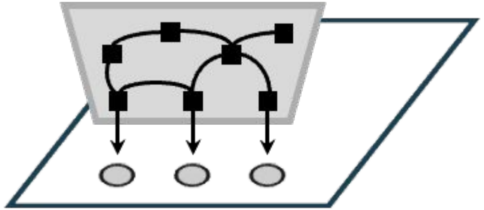

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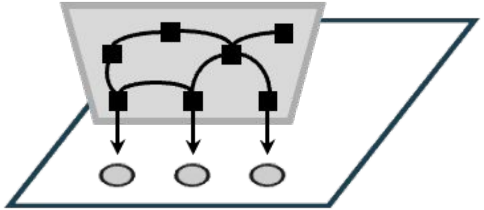

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Can we break up the problem?

Outline

- Background and Motivation
  - Quantum LDPC Codes
  - Code Surgery Methods
- Auxiliary Graph Surgery on QLDPC Codes
  - Graph Desiderata
  - Universal Adapter for Joint-measurements
- Case Study: [[144,12,12]] Bivariate bicyclic code

Joint-measurements: a modular approach

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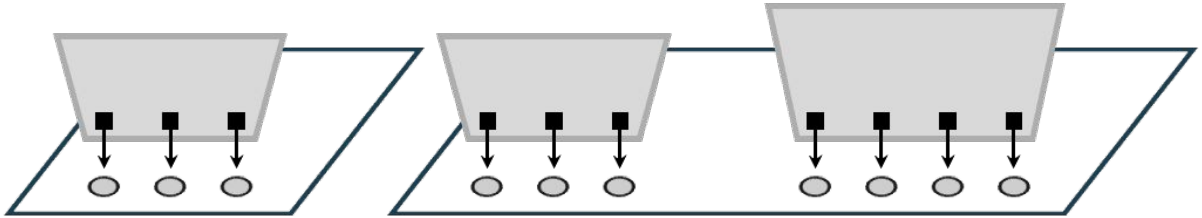

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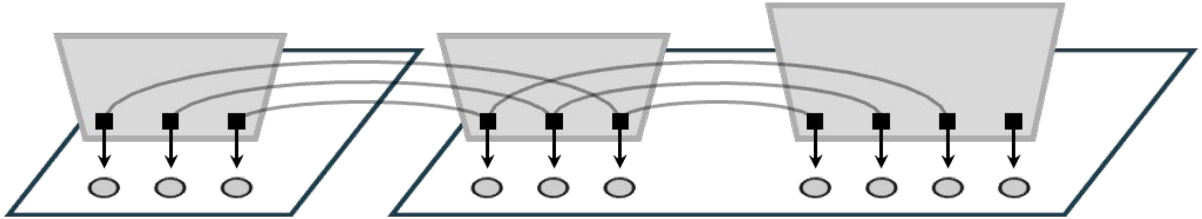

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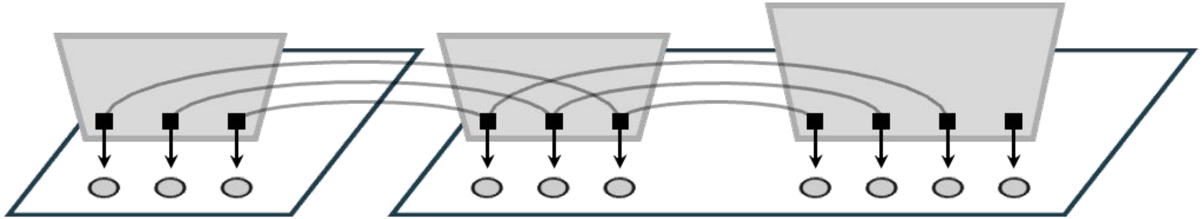

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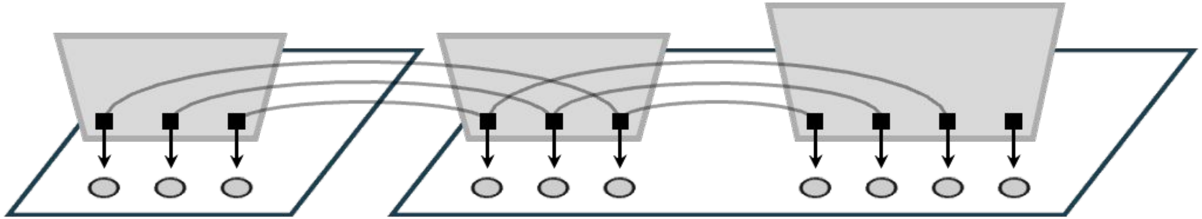

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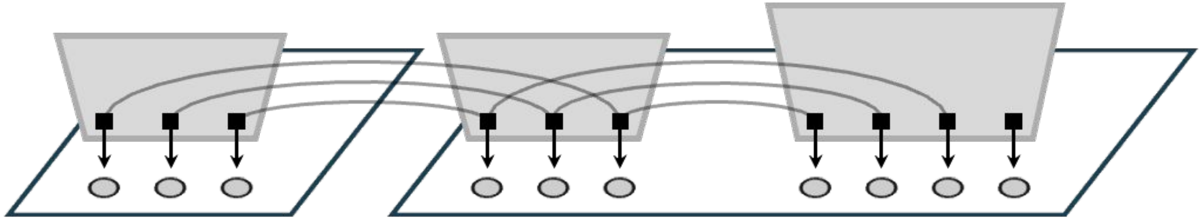

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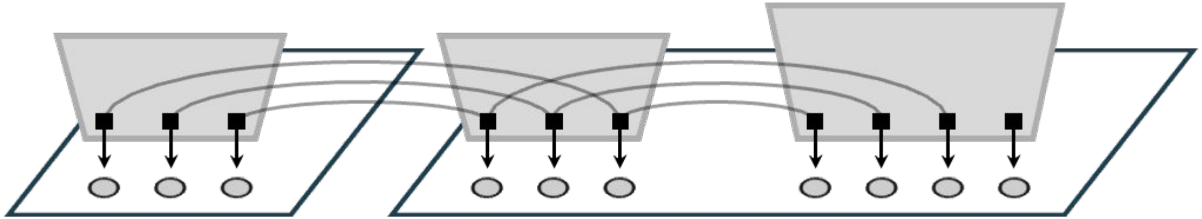

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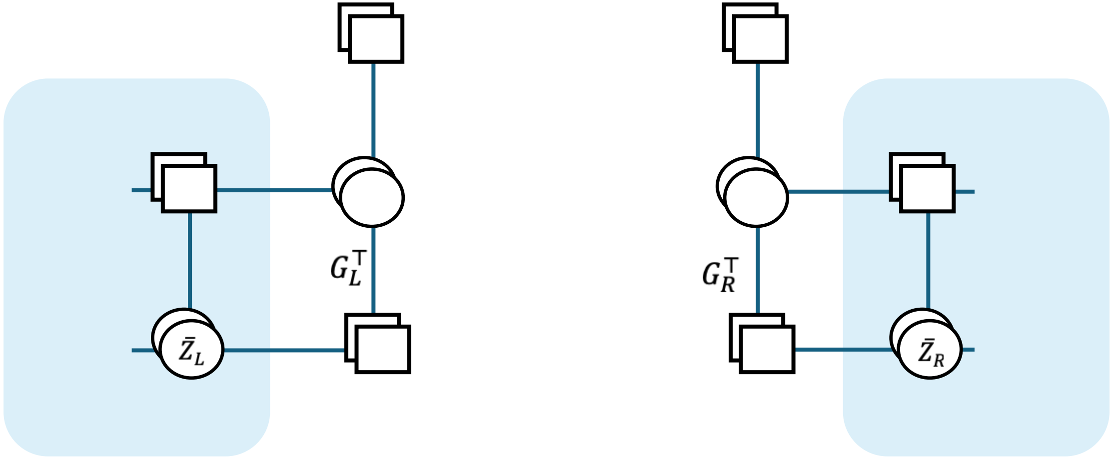
New cycles are at most length 8 ≡ new X stabilizers are at most weight 8

Universal adapter: a new primitive



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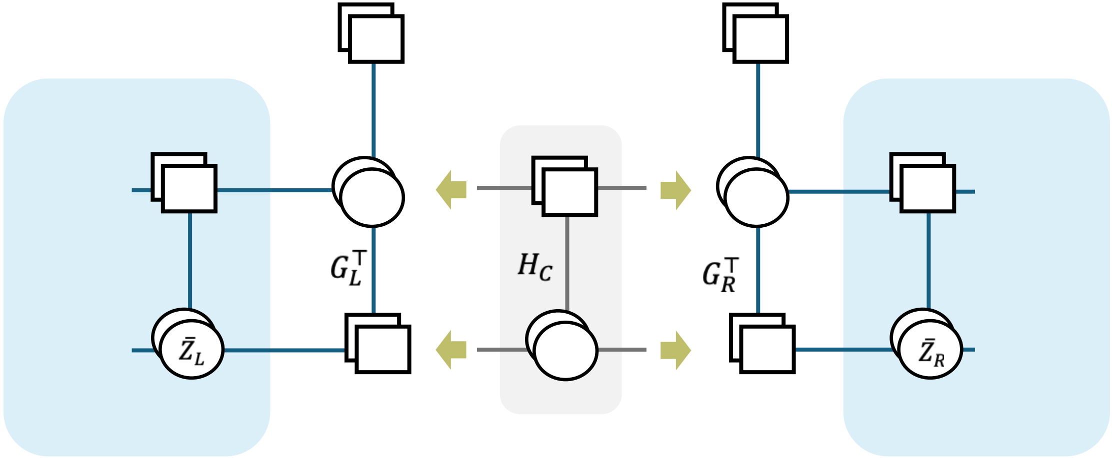

Key insight: any tree-like graph is equivalent to a repetition code up to a transformation

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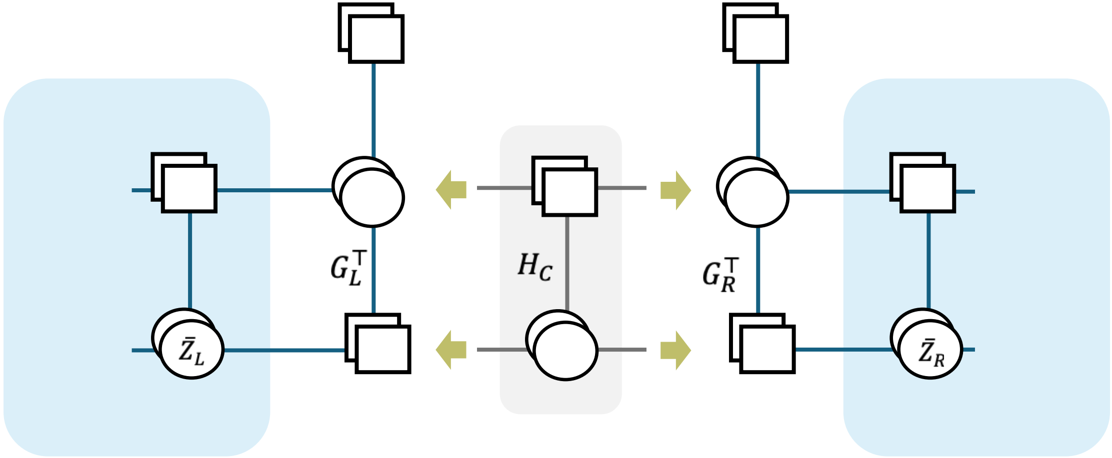

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[Universal Adapters between quantum LDPC codes], 2410.03628

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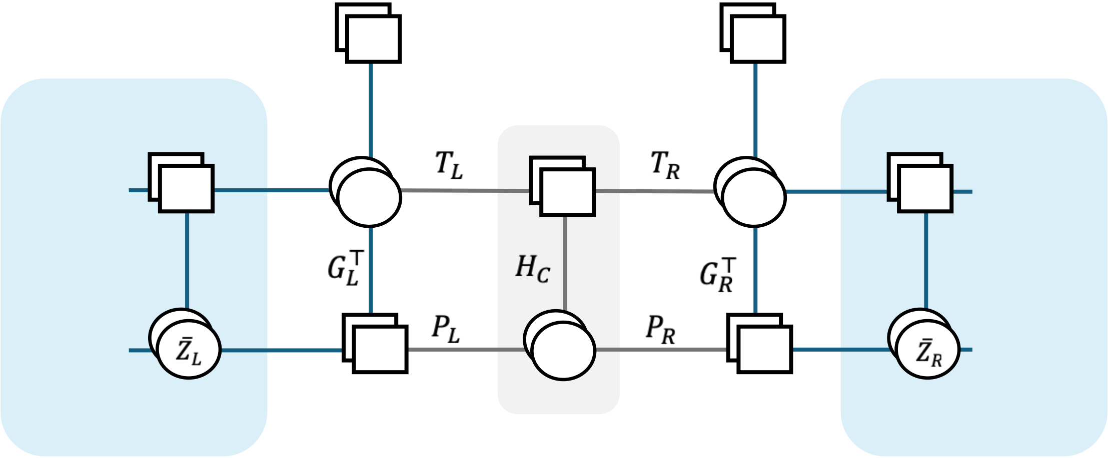

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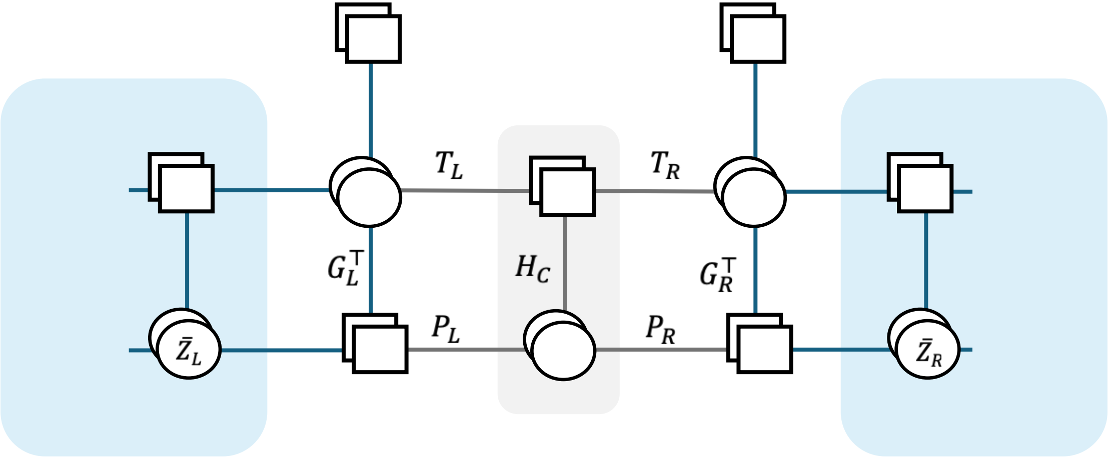


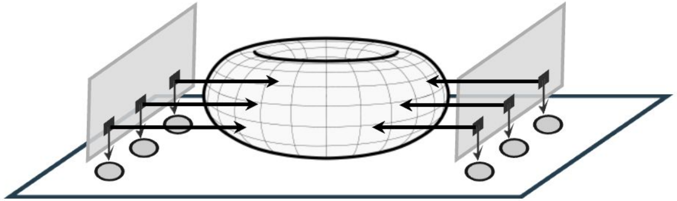
Significance : These can be arbitrary* logical operators in the LDPC code, they could belong to the same or different codeblocks, or even different quantum codes

A "Universal" way to connect between any two logical operators in quantum LDPC codes

Universal adapter: Connecting different qLDPC codes

- Arbitrary joint-measurements in the same or different codeblocks
- **multi-code architectures**…

Can be useful for teleportation, transversal gates, magic state factory, code-switching…



Can leverage known symmetries in codes, and implement these gates on logical qubits of other codes.

In Summary

- **Addressable** gates for multi-qubit code blocks
- A "**universal**" scheme applicable to **arbitrary** quantum LDPC codes
- Space overhead is only ~**linear** - which is a quadratic improvement over previous schemes; the LDPC overhead advantage is now beginning to show even for logical gate schemes.

- Can connect arbitrary codes for **multi-code architectures** with just ~**d extra qubits** allowing one to leverage unique advantages of different codes, while keeping architecture LDPC.

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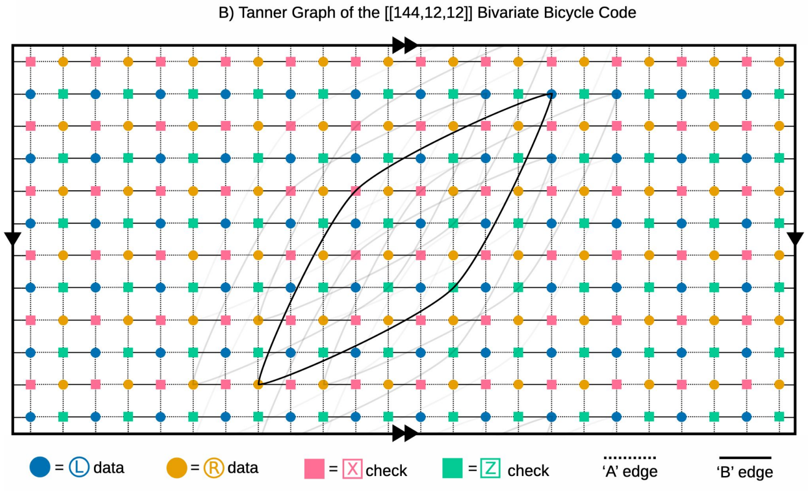

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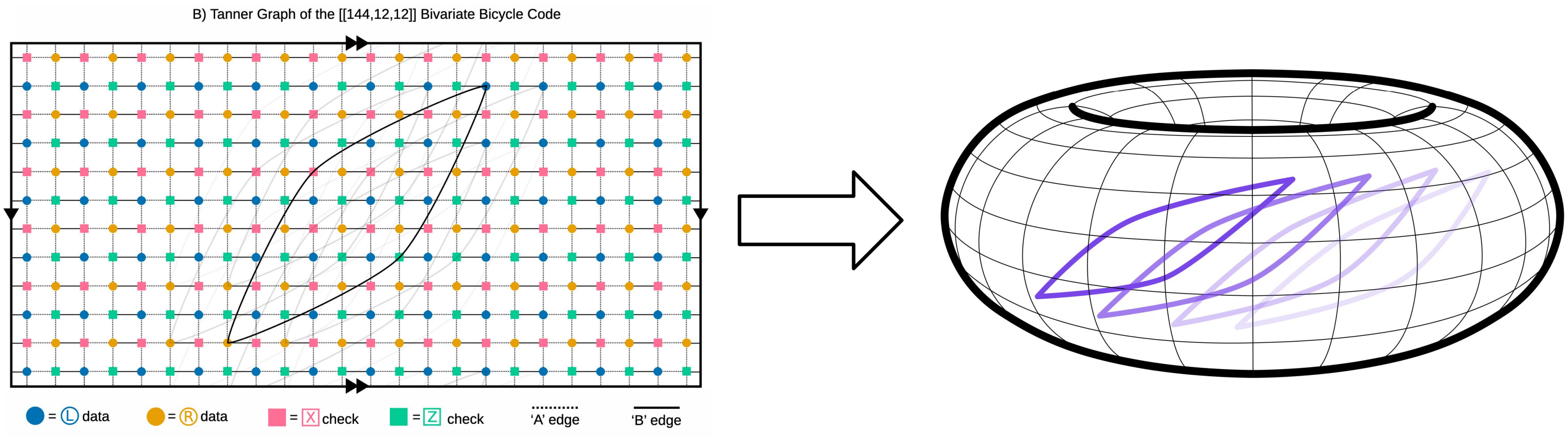

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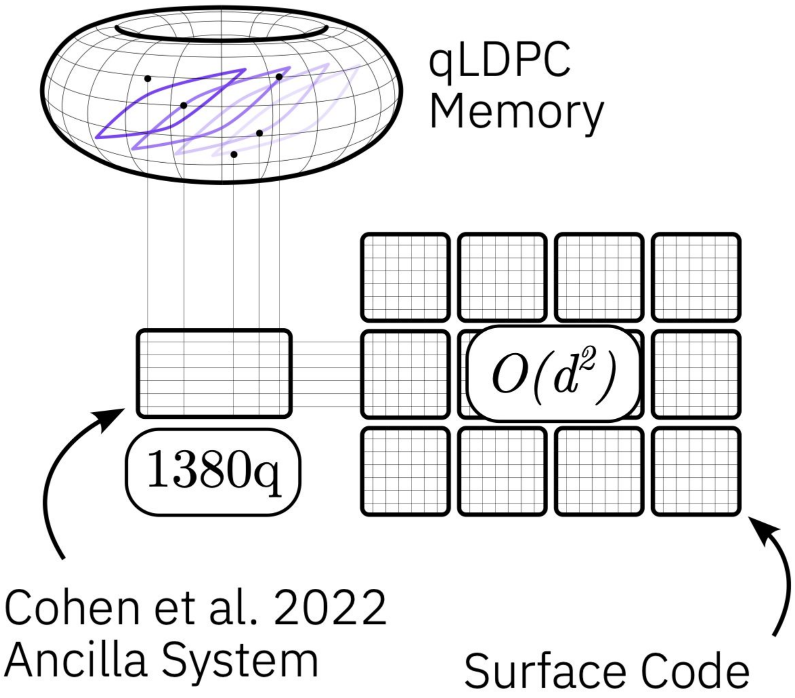

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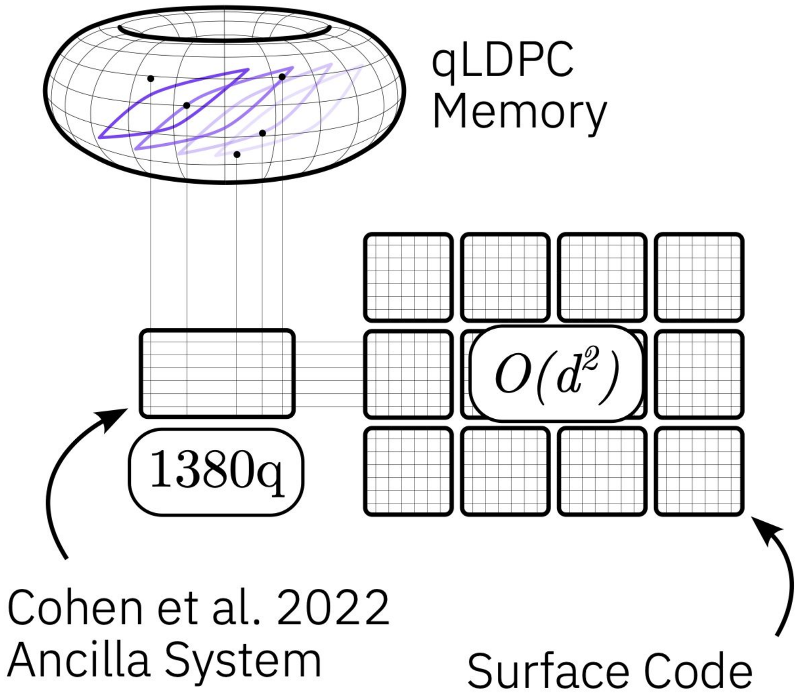

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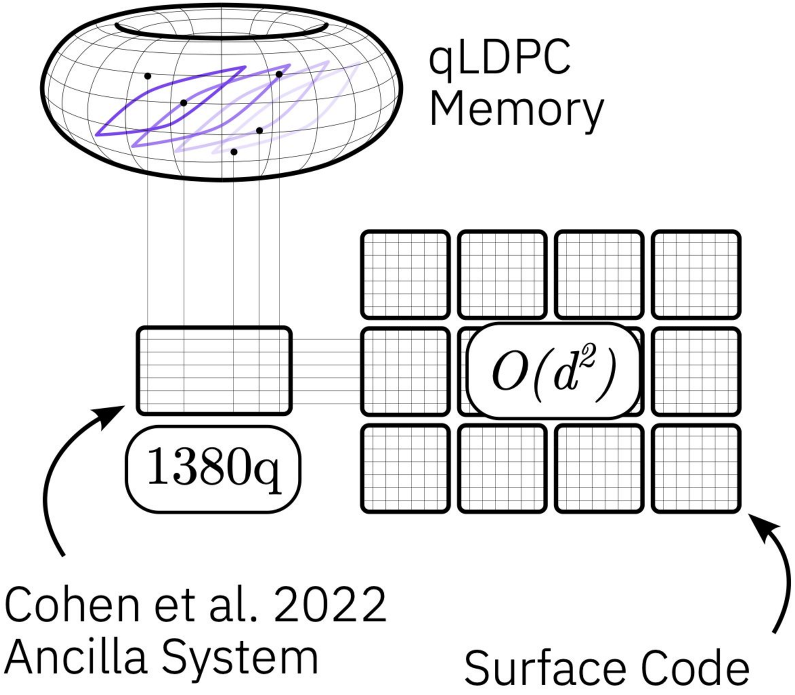

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Teleport logical information into a surface code patch to do computation.

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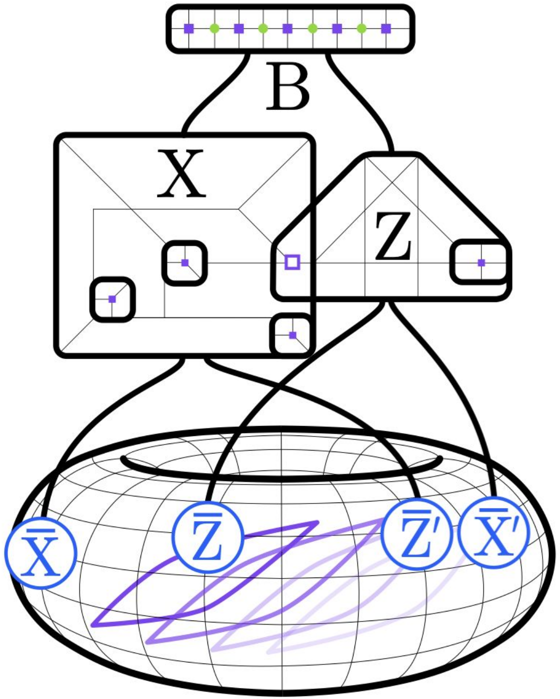
New Proposal for Logical Computation



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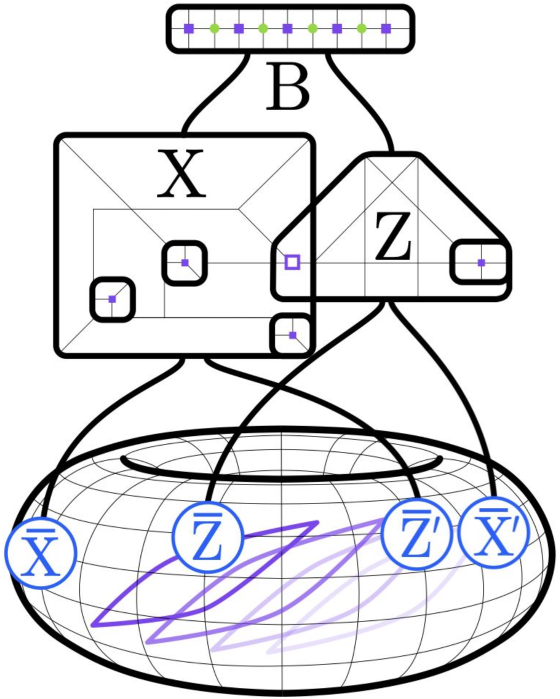

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Use a bridge to perform joint measurement.

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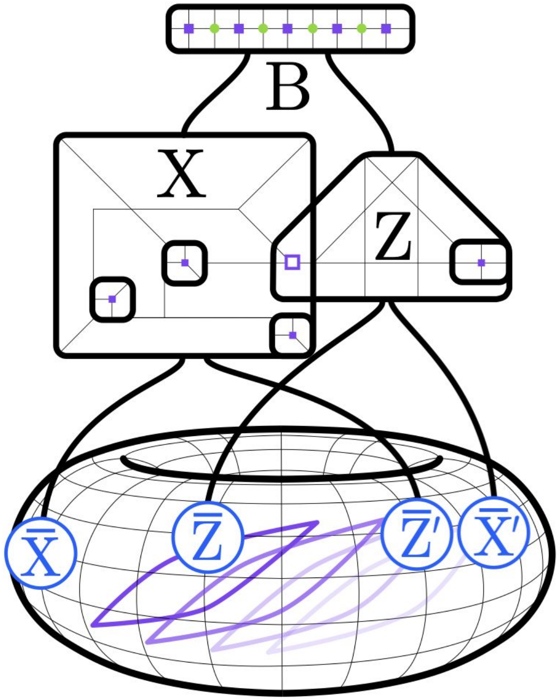

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Automorphism gates + logical measurement =

Full logical Clifford group!

Promising numerical benchmarking.

Logical Computation on QLDPC Codes through Surgery

With Case Study on Bivariate Bicycle codes

Andrew Cross, **Zhiyang He (Sunny)**, Tomas Jochym-O'Connor, Patrick Rall, **Esha Swaroop**, Dominic Williamson, Theodore Yoder

The End