Single-Shot Decoding of Good Quantum LDPC Codes

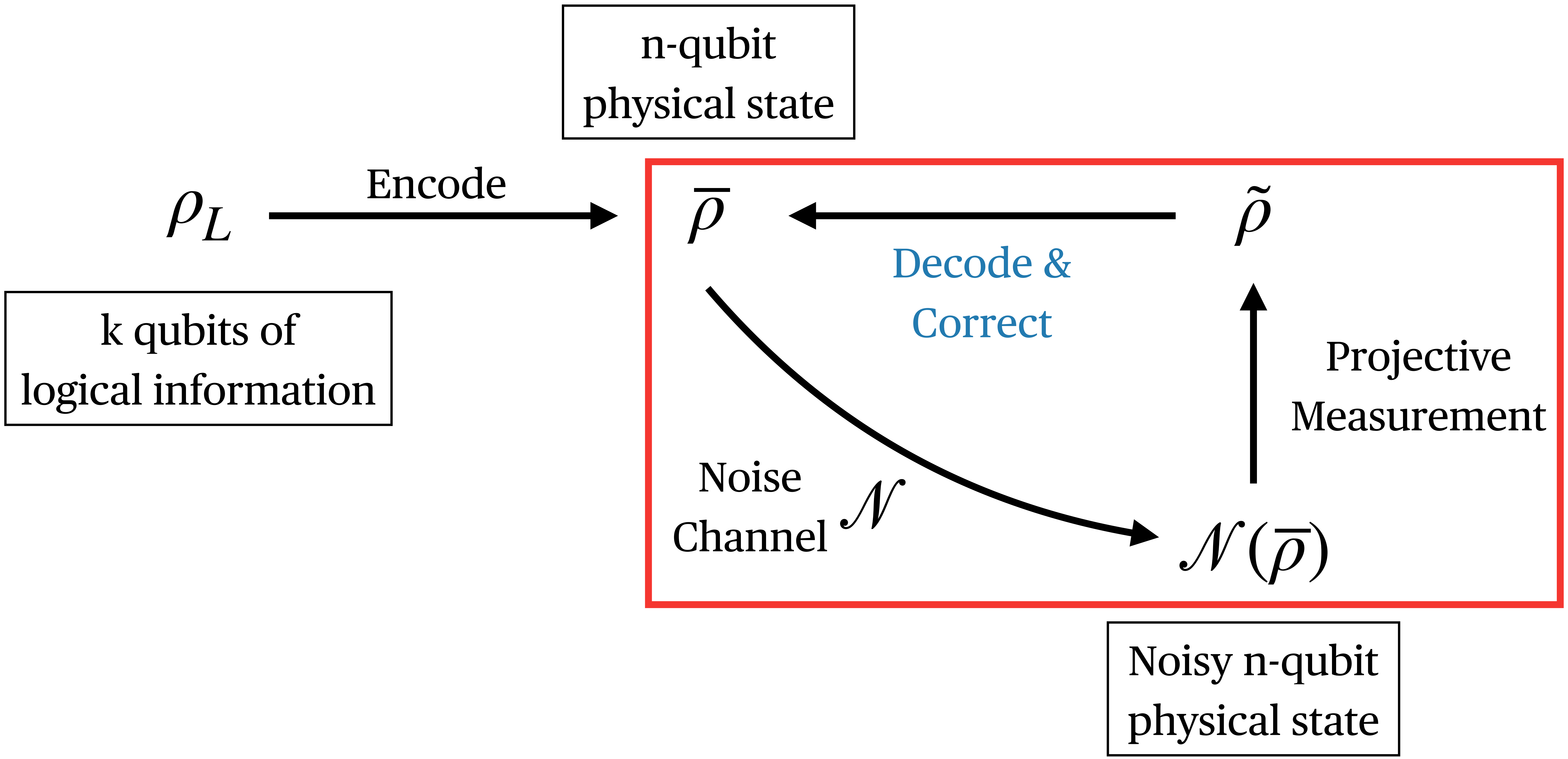
Protecting Memory with Constant-Time Decoding

Shouzhen Gu, Eugene Tang, Libor Caha, Shin Ho Choe, **Zhiyang He (Sunny)**, Alesander Kubica

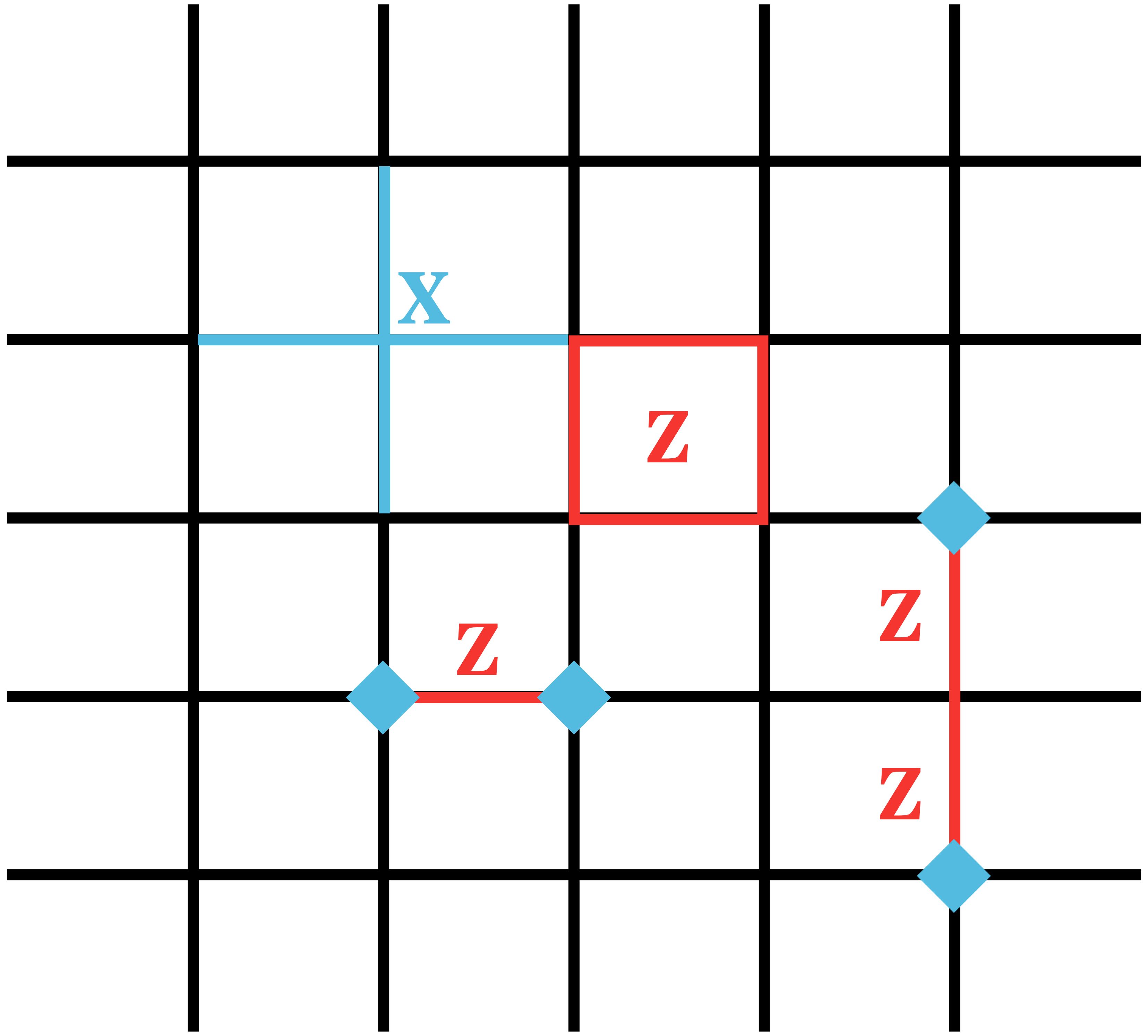
Single-Shot Decoding of Good Quantum LDPC Codes

What do all of these words mean?

Quantum Codes



Quantum Codes

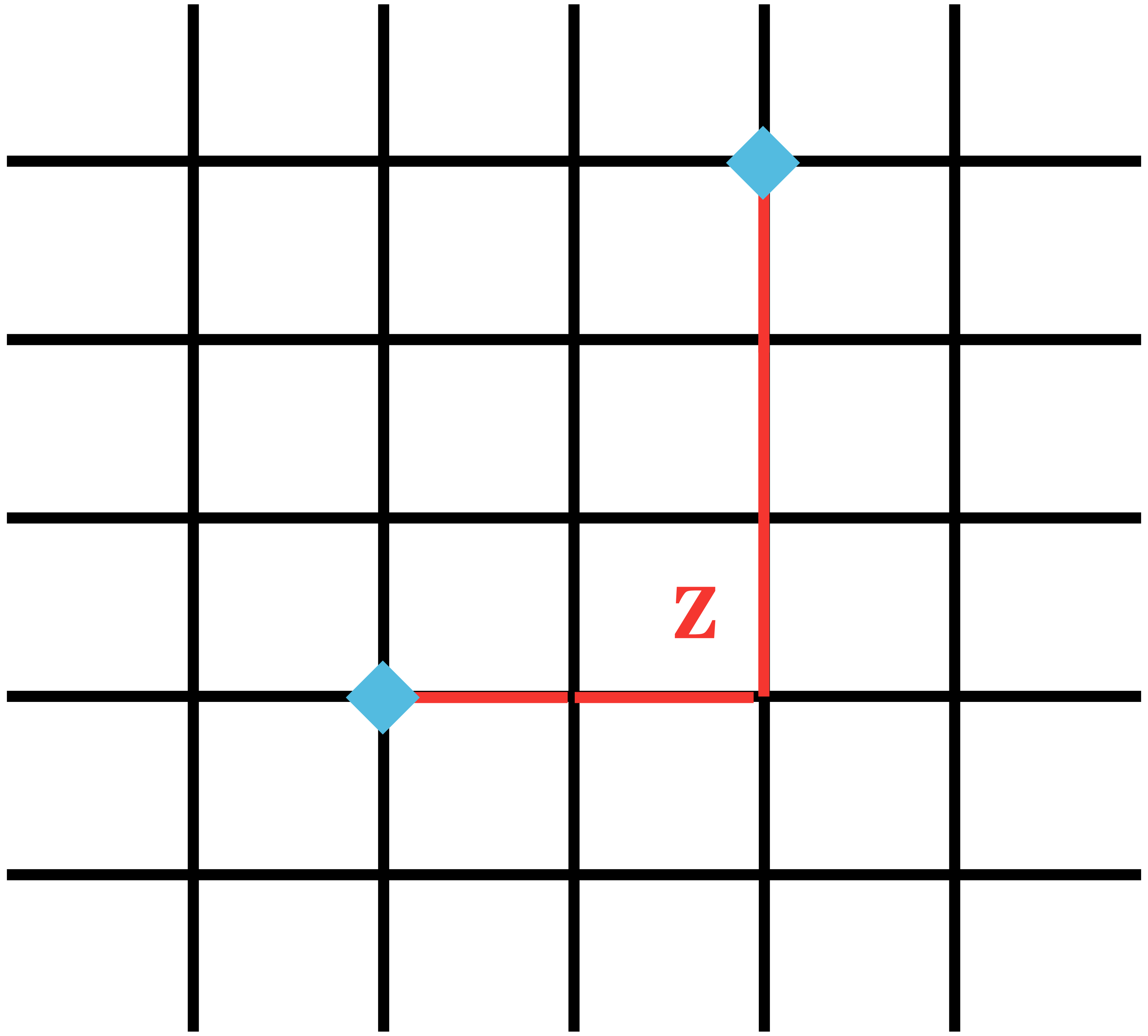


Toric Code

Every edge is a qubit,
Every vertex is a X-check,
Every square is a Z-check.

A Z-error on a qubit triggers the X-checks on the endpoints.

Decoding of Quantum Codes

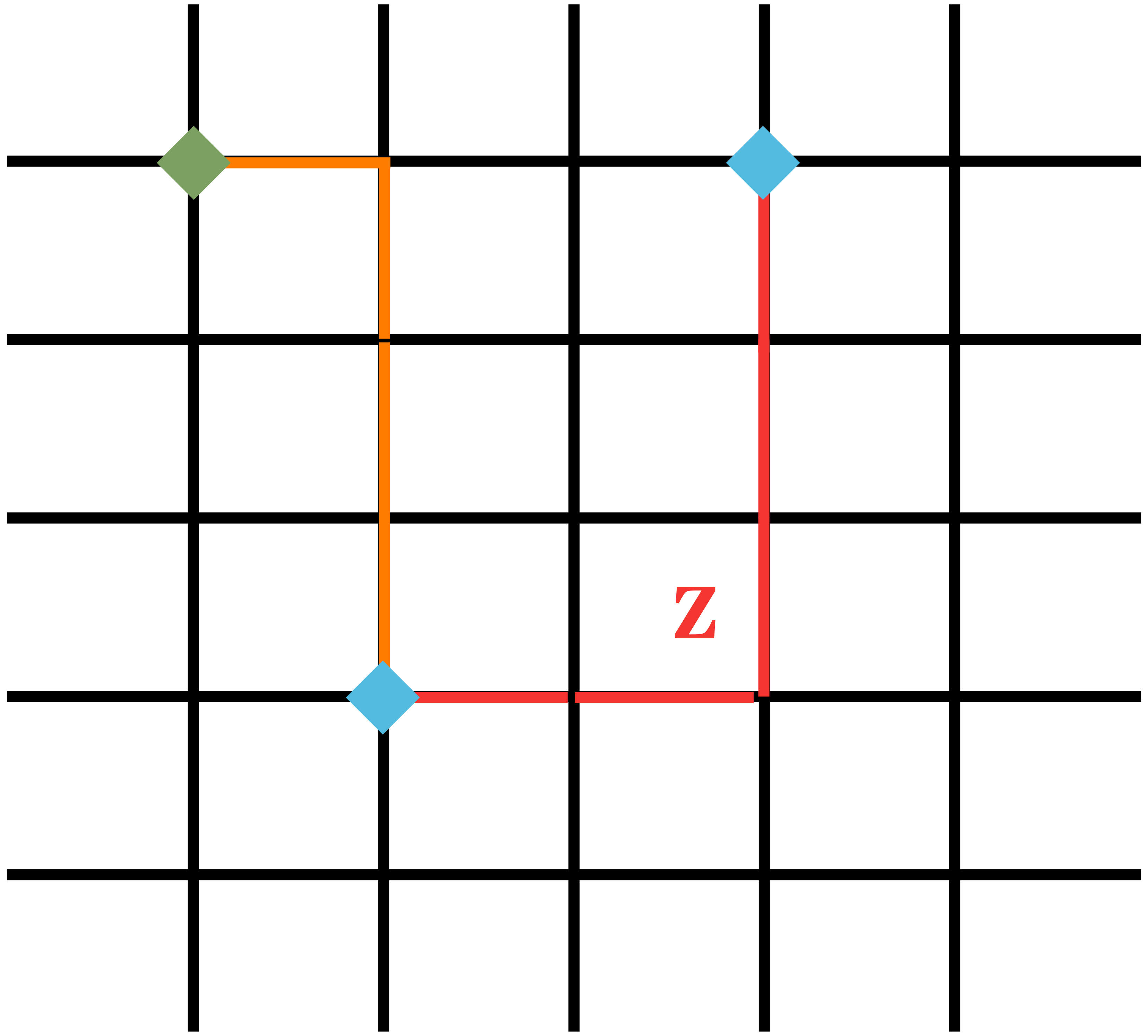


If we observe some X-syndrome, what is the most likely Z-error that occurred?

On the Toric code, this decoding problem is fairly simple: match the syndromes together!

But what if our syndromes could be wrong too?

Decoding of Quantum Codes

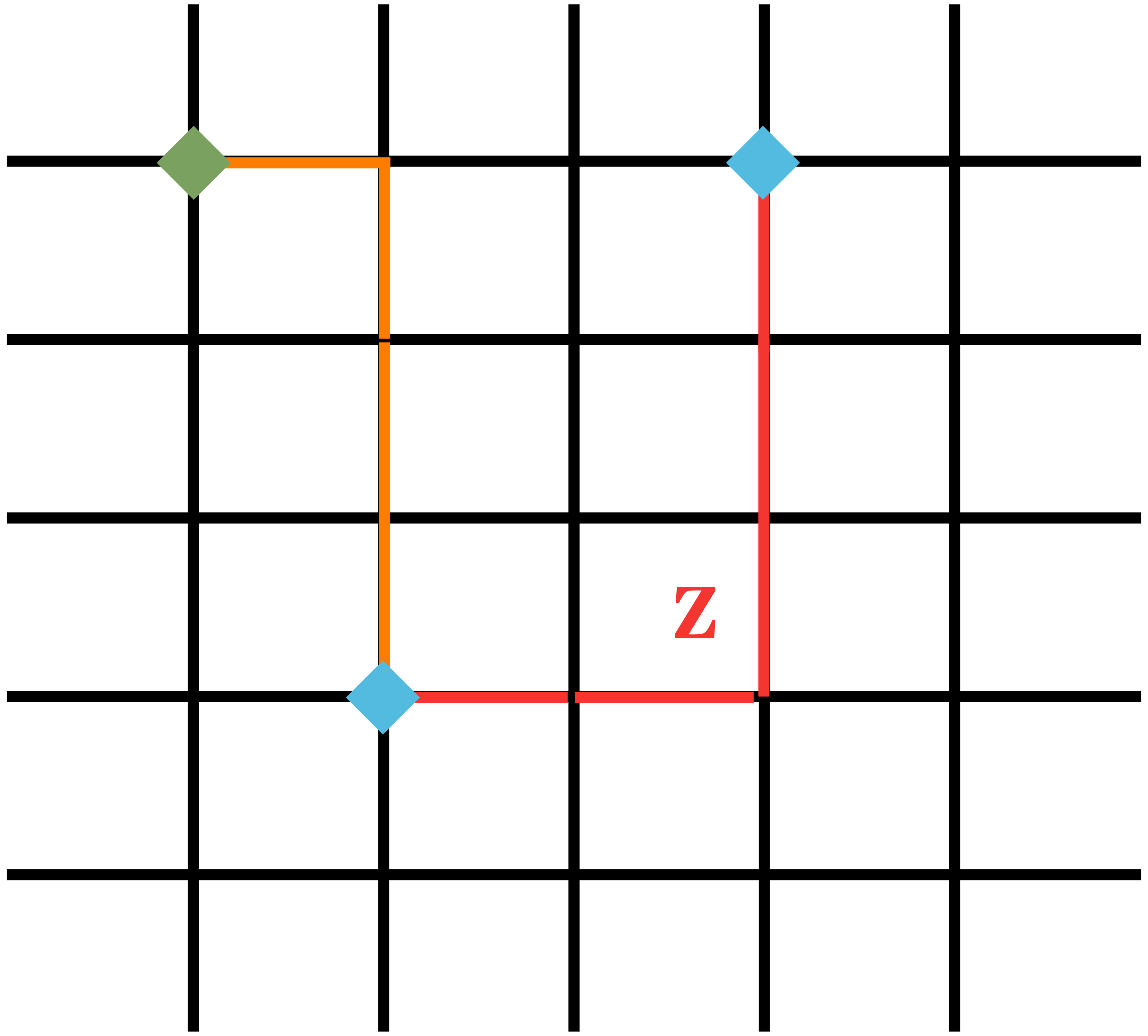


Phenomenological Noise Model:
Pauli errors on qubits, flip error on syndromes.

If a true syndrome disappeared and a fake syndrome appeared, our correction can be completely wrong.

What can we do?

Decoding of Quantum Codes



Natural Idea: Repeat measurements for d rounds to catch measurement errors.

Advantage: Very effective, widely used;

Disadvantage:

1. O(d) quantum time overhead,
2. Decoding is often much slower.

Idea: can we design our code to protect against measurement errors?

Single-Shot Decoding

Single-shot Decoding: under phenomenological noise, given one (1) round of measured noisy syndrome, can we decode so that the residue error is small?

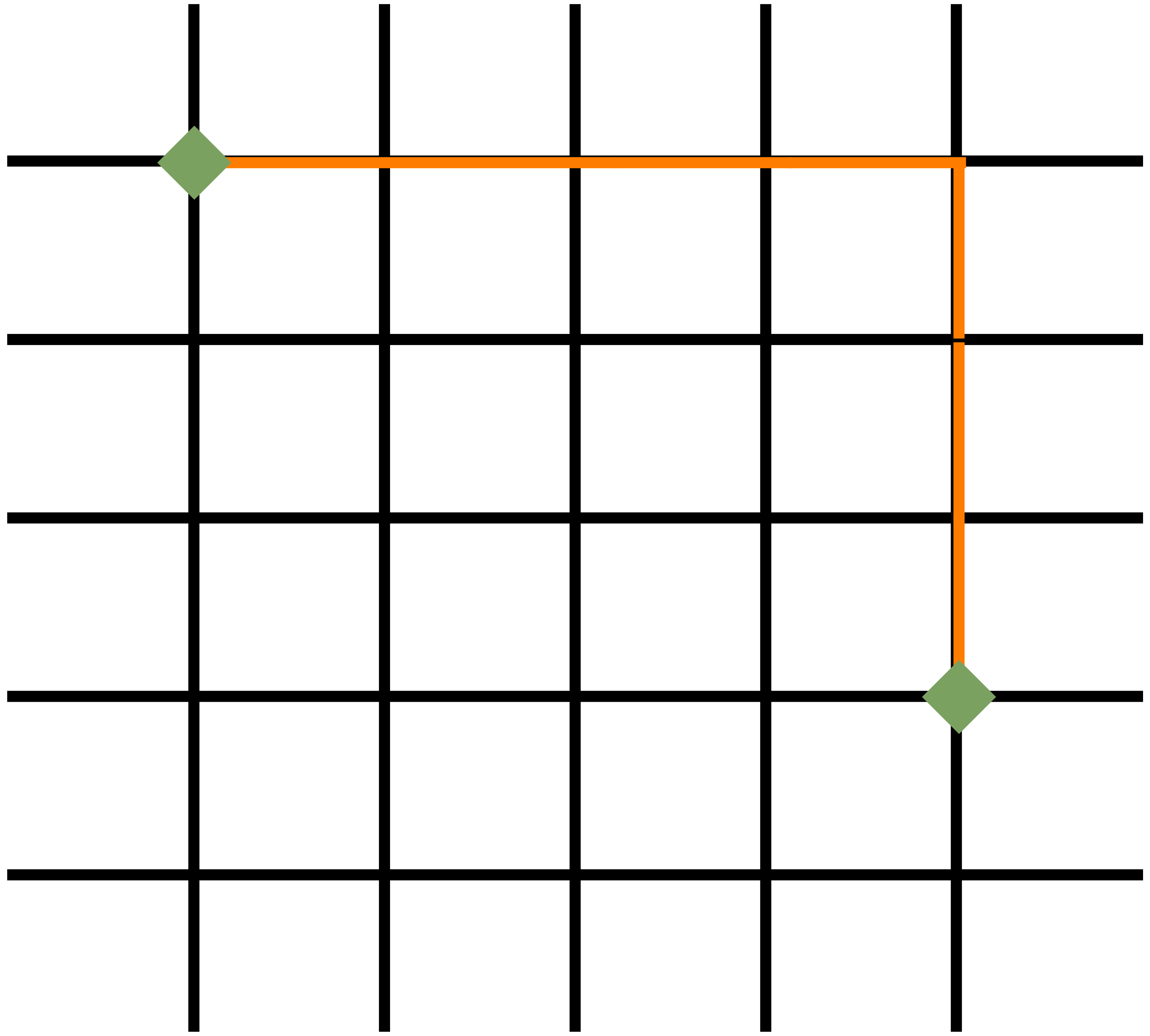
=> reduces time overhead by a factor of d!

Q: What's the obstacle between many codes and single-shot decodability?

A: Existence of large errors with small, uncorrectable syndromes.

We know two methods to overcome this obstacle:

1. Local redundancy in stabilizer checks;
2. Expansion of Tanner graph.



Single-Shot Decoding

Existing codes with single-shot decoders:

- Topological Codes:
  - 4D toric code [BDMT17], 3D subsystem toric code [KV22], 3D gauge color code [Bom15]
  - These codes has local redundant checks, so we can detect & correct measurement errors locally.
- Expansion Based QLDPC Codes:
  - Quantum expander codes [FGL18]
  - Quantum Tanner codes [This work]
  - Expansion ensures that errors correctable by the decoder triggers many syndromes.
- See also [Cam19], which formulates and constructs redundant checks as ‘meta-checks’.

Good Quantum LDPC Codes

Quantum LDPC Codes: All stabilizer checks have weight O(1), all qubits are touched by O(1) checks.

Equivalently: The check adjacency graph of the code has constant degree.

Good QLDPC Codes: [n, k, d] code – # of physical qubits = n, # of logical qubits = k, distance = d.

A code is asymptotically good if $k, d = \Omega(n)$.

There are three constructions based on the same underlying ideas: [PK22], [LV22], [DHLV22]

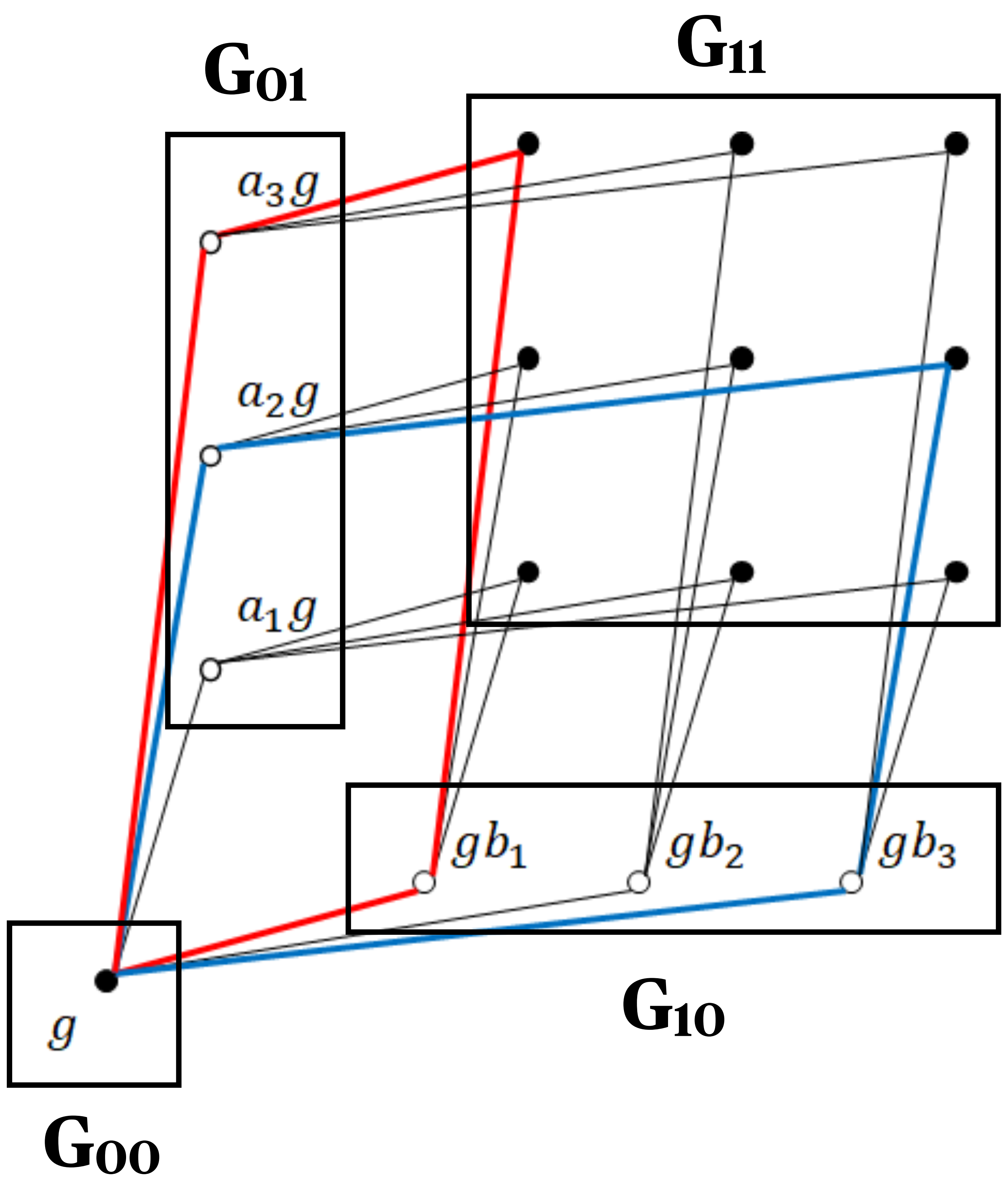
We focus on Quantum Tanner codes [LV22].

Good Quantum LDPC Codes

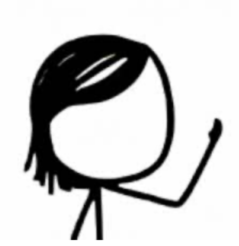
I love the left-right Cayley complex! How can I build it at home?

1. Consider a group G and two generating sets, A and B.
2. Take 4 copies of the group G (drawn and placed disproportionally for artistic purposes)
3. Create A edges: For every $a \in A$, for every $g \in G_{00}$ or G_{10}, connect it to $ag \in G_{01}$ or G_{11}.
4. Create B edges: For every $b \in B$, for every $g \in G_{00}$ or G_{01}, connect it to $gb \in G_{10}$ or G_{11}.

Great job! Enjoy your own left-right Cayley complex.



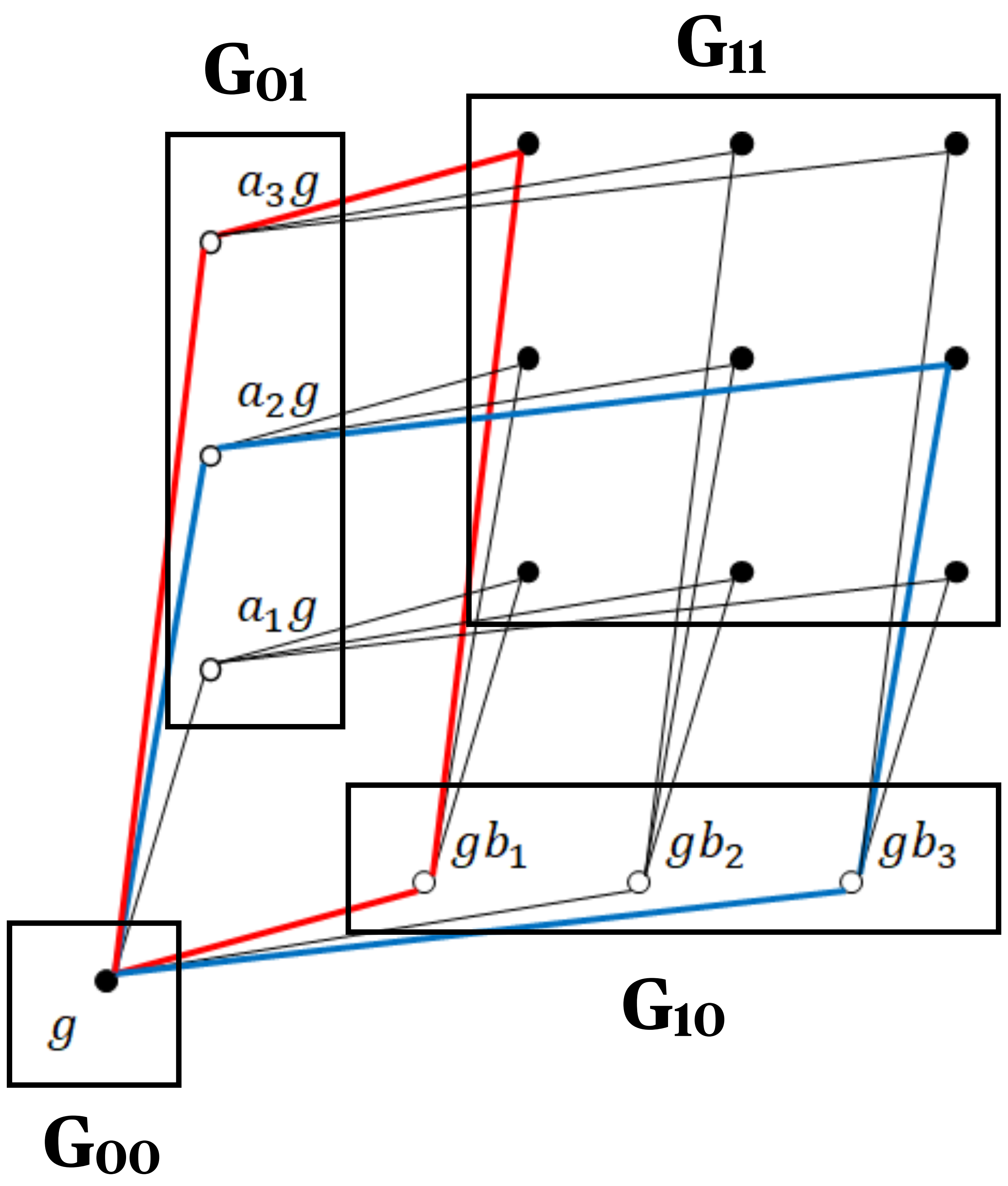
Good Quantum LDPC Codes



What about Quantum Tanner codes?

1. Take the left-right Cayley complex we just built, and place a qubit on every square.
2. Now every vertex touches a total of $|A| \cdot |B|$ many such squares. On every vertex $g \in G_{00}$ and G_{11}, place some X-checks which acts on the incident squares.
3. Build Z-checks on $g \in G_{01}$ and G_{10} similarly.

You just built an asymptotically good code!



Decoding of Good Quantum LDPC Codes

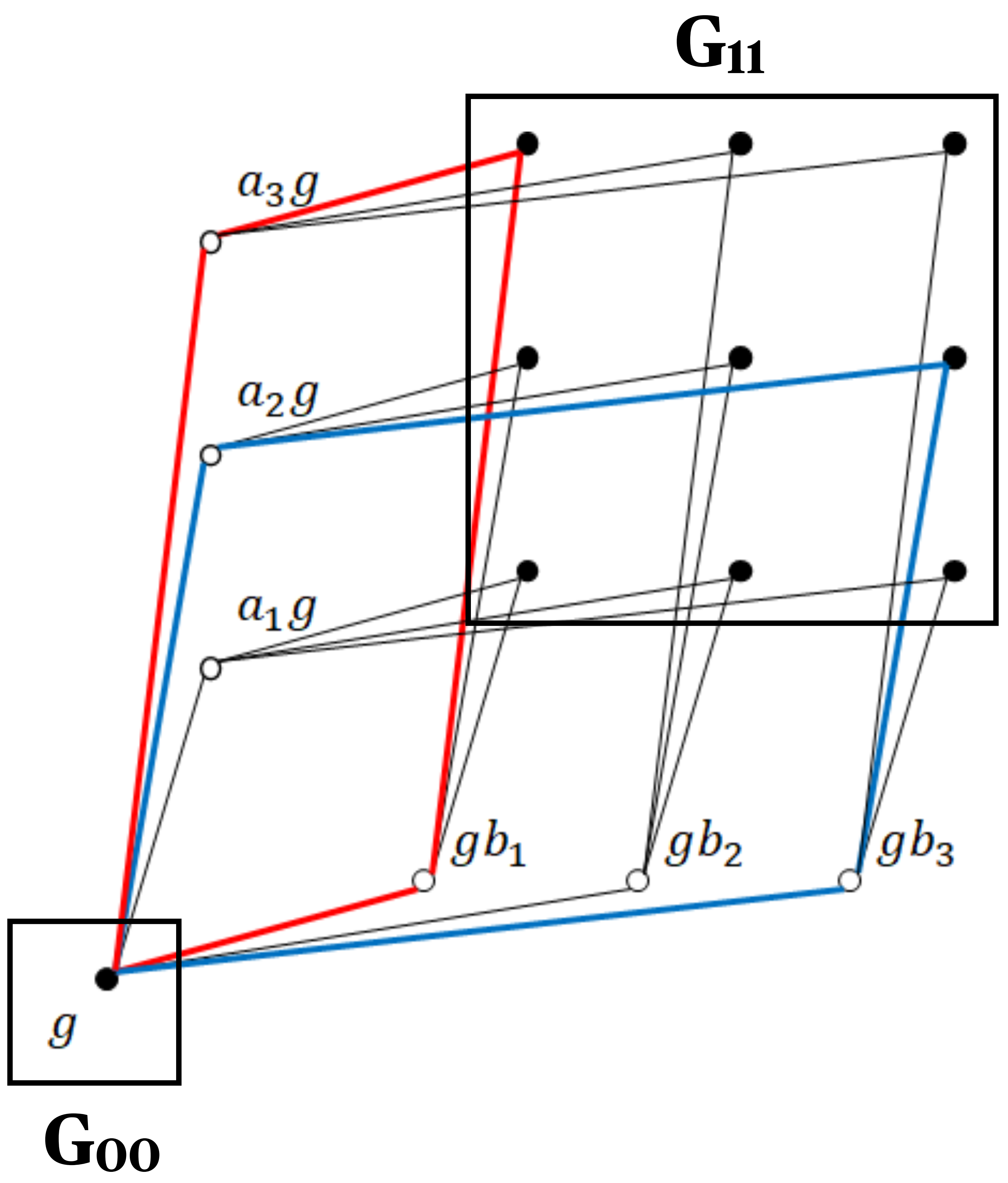
Decoding problem: given syndromes on vertices in G_{00} and G_{11}, find squares to flip to correct them.

Observe: every vertex $g \in G_{00}$ touches disjoint set of squares, so we can easily find correction C_{00} that corrects all syndromes in G_{00}. Note that C_{00} does NOT correct syndromes in G_{11}!

Similarly find correction C_{11} for G_{11}. Define the mismatch vector as

$$Z = C_{00} \oplus C_{11} \in \mathbb{F}_2^n.$$

If Z is zero, then $C_{00} = C_{11}$ and we found a valid correction! This decoder is proposed by [LZ22].

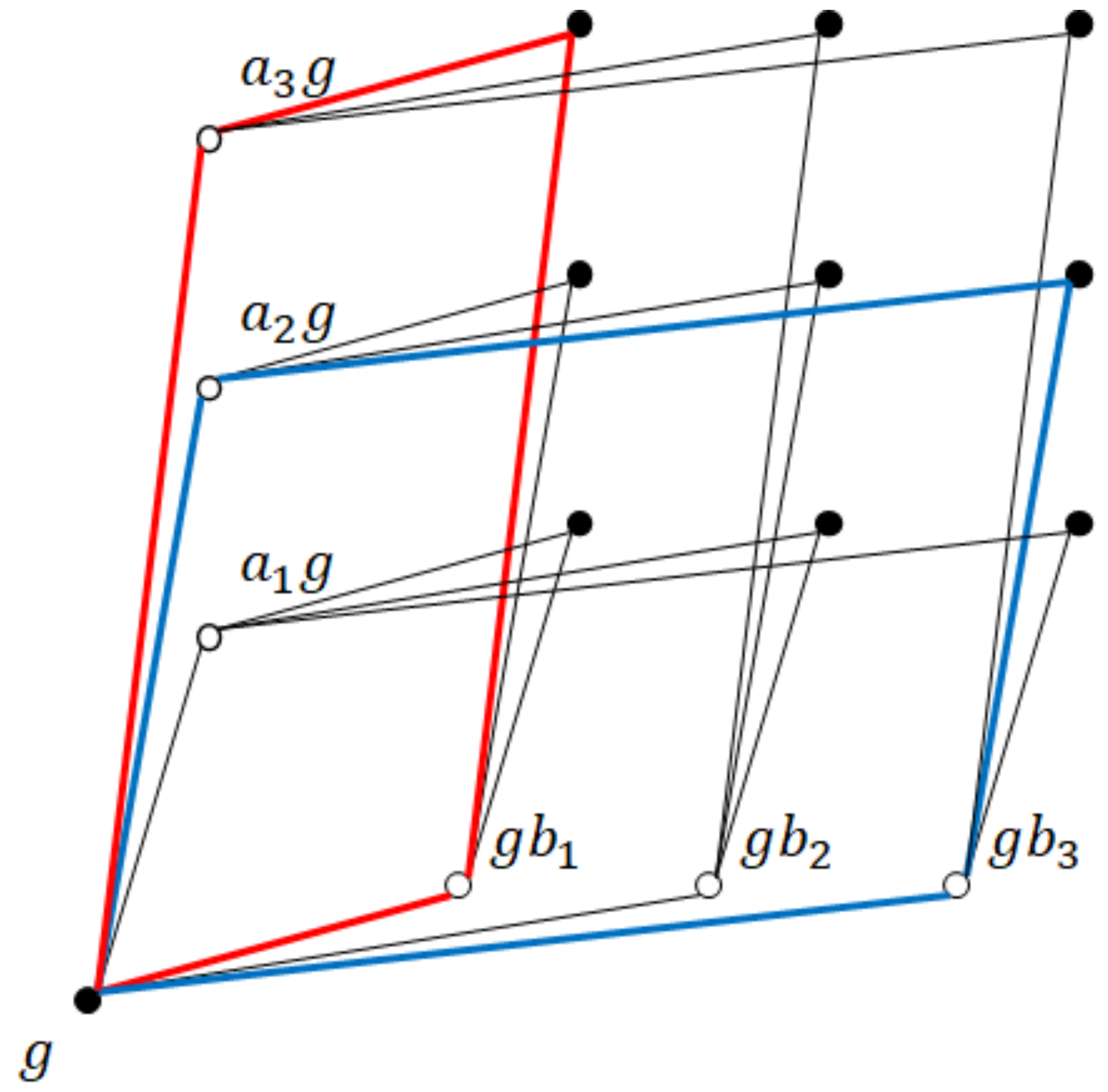


Decoding of Good Quantum LDPC Codes

Decoding idea: Greedily find vertices g, and corrections within g's neighborhood to reduce weight of mismatch.

Why does a greedy algorithm work? Intuitively,

- Expansion of complex => there always exists a vertex g that has too much mismatch.
- Robustness of local code => we can find a good flip within g's neighborhood.



Single-Shot Decoding of Good Quantum LDPC Codes

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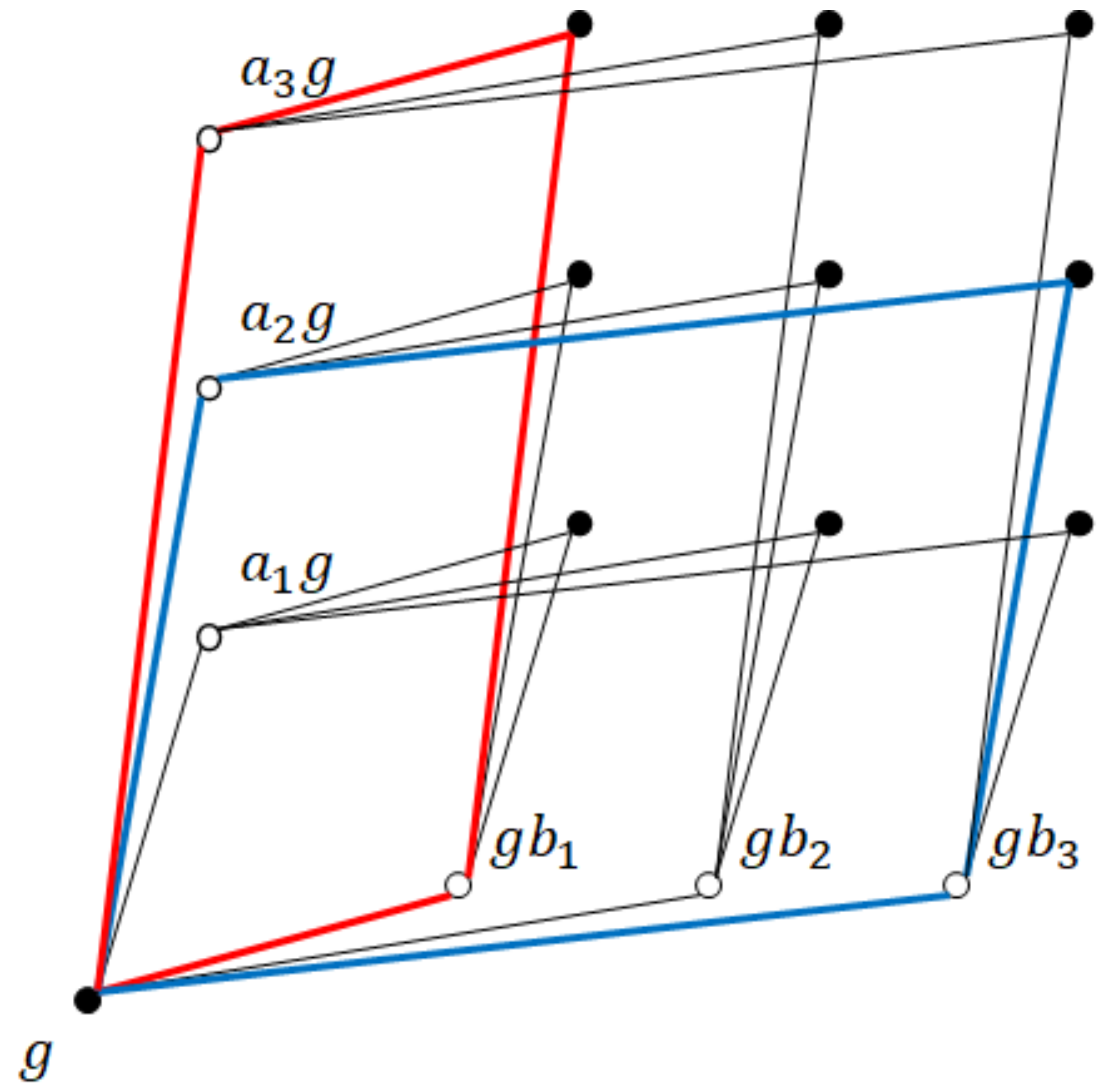
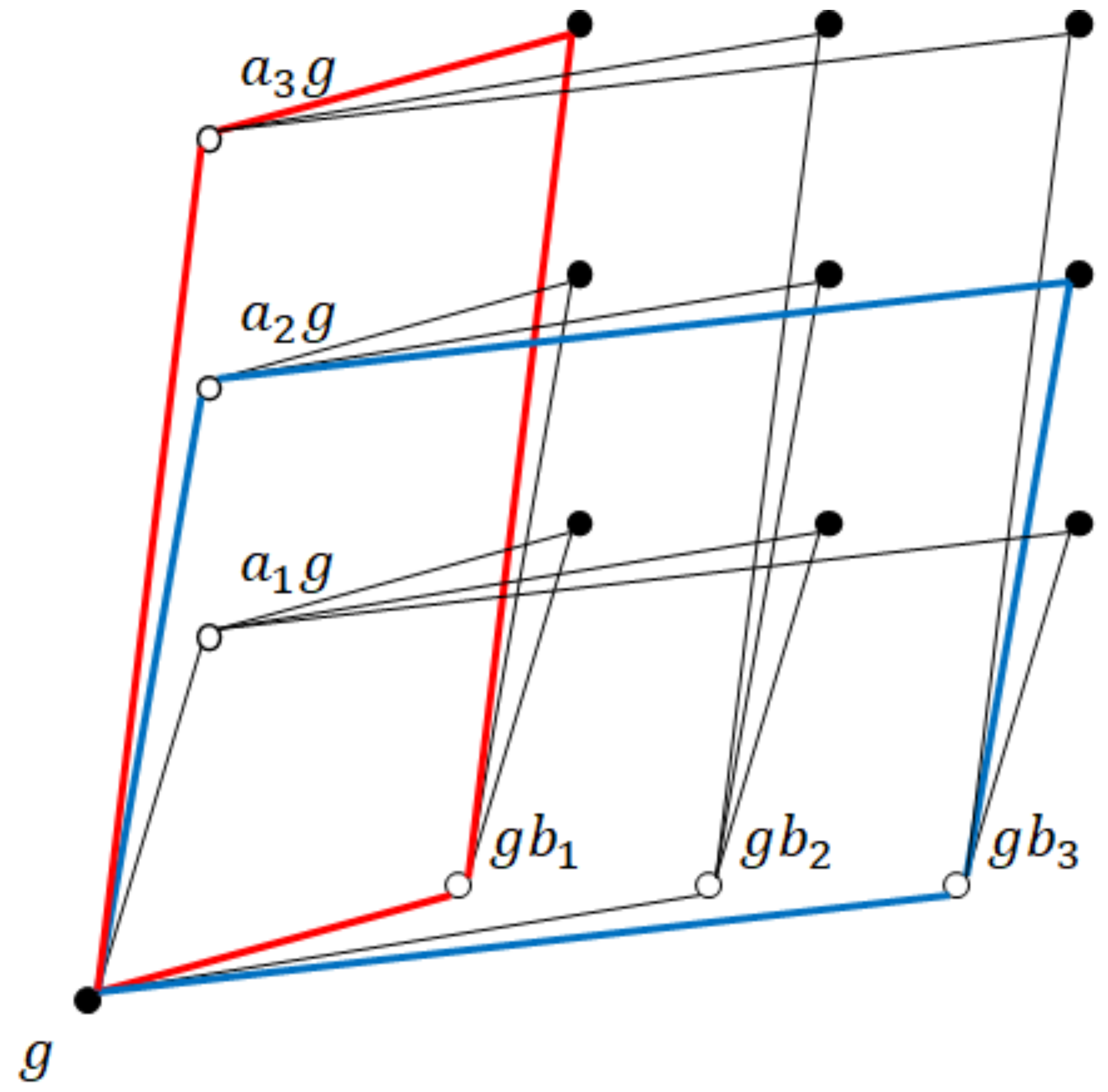


Single-Shot Decoding of Good Quantum LDPC Codes

What if there are measurement errors?

- Greedy, local decoder => measurement error is decomposed into local errors on mismatch vector.
- Expansion of complex => under such local errors, there always exists a vertex g that has too much mismatch.
- Robustness of local code => we can find a good flip within g's neighborhood.



Single-Shot Decoding of Good Quantum LDPC Codes

Main Results 1: Sequential Single-Shot Decoder (informal)

For the quantum Tanner code, the sequential, linear time decoder of [LZ22], with slight modifications, is single-shot in the following sense:

Denote qubit error as e, syndrome error as D. Suppose e, D are arbitrary errors* with linearly bounded weight: $|e|, |D| \leq O(n)$. Then there exists constant β such that

$$\text{Residue error weight} \leq \beta |D|.$$

*Arbitrary error covers stochastic errors too.

Single-Shot Decoding of Good Quantum LDPC Codes

Main Results 2: Parallel Single-Shot Decoder (informal)

For the quantum Tanner code, the parallel decoder of [LZ22], with slight modifications, is single-shot:

Suppose e, D are arbitrary errors with linearly bounded weight. There exists constant β such that if we run the parallel decoder for O(t) rounds,

$$\text{Residue error weight} \leq 2^{-\Omega(t)}|e| + \beta|D|.$$

For best decoding result, take $t = \log(n)$. Each round takes constant time.

Single-Shot Decoding of Good Quantum LDPC Codes

Main Results 3: Memory protected by constant-time decoding

Given a decoder with our single-shot properties, a quantum memory under bounded noise can be protected for an exponential number of single-shot QEC rounds. If the noise is stochastic, the memory has a threshold.

Specifically, the constant-round parallel decoder of Main Result 2 suffices. A memory based on quantum Tanner codes has constant time-overhead, except for a final round of $O(\log n)$ time decoding.

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