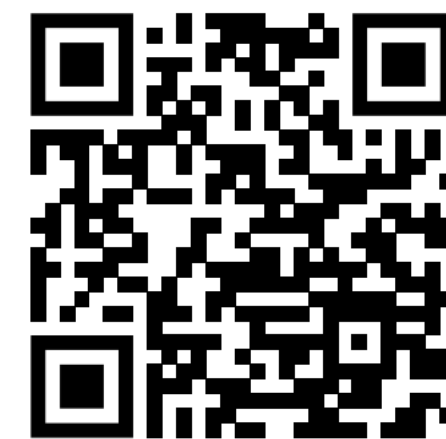


# Constant-Time Surgery on QLDPC Codes

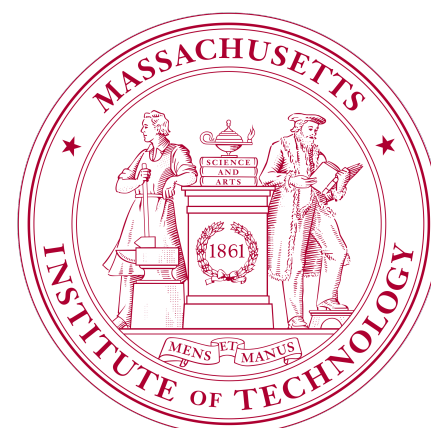
QEC @ Santa Barbara, June 11, 2026

**Katie Chang**, Alexander Cowtan, Sunny He,  
Tomas Jochym-O'Connor, Dominic Williamson, Theodore Yoder, Guanyu Zhu

[2510.14895] [2603.02157]

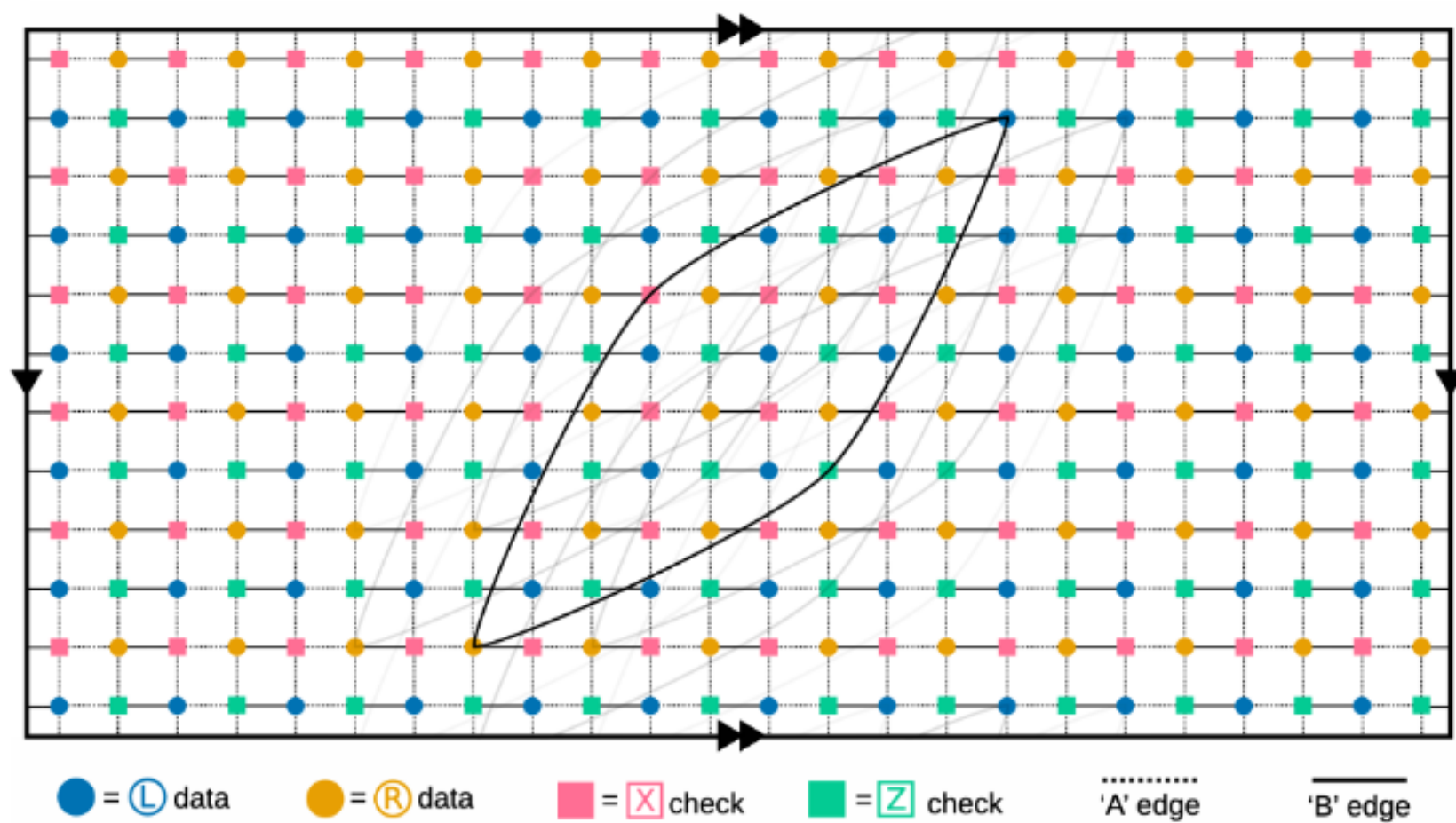


Slides here!

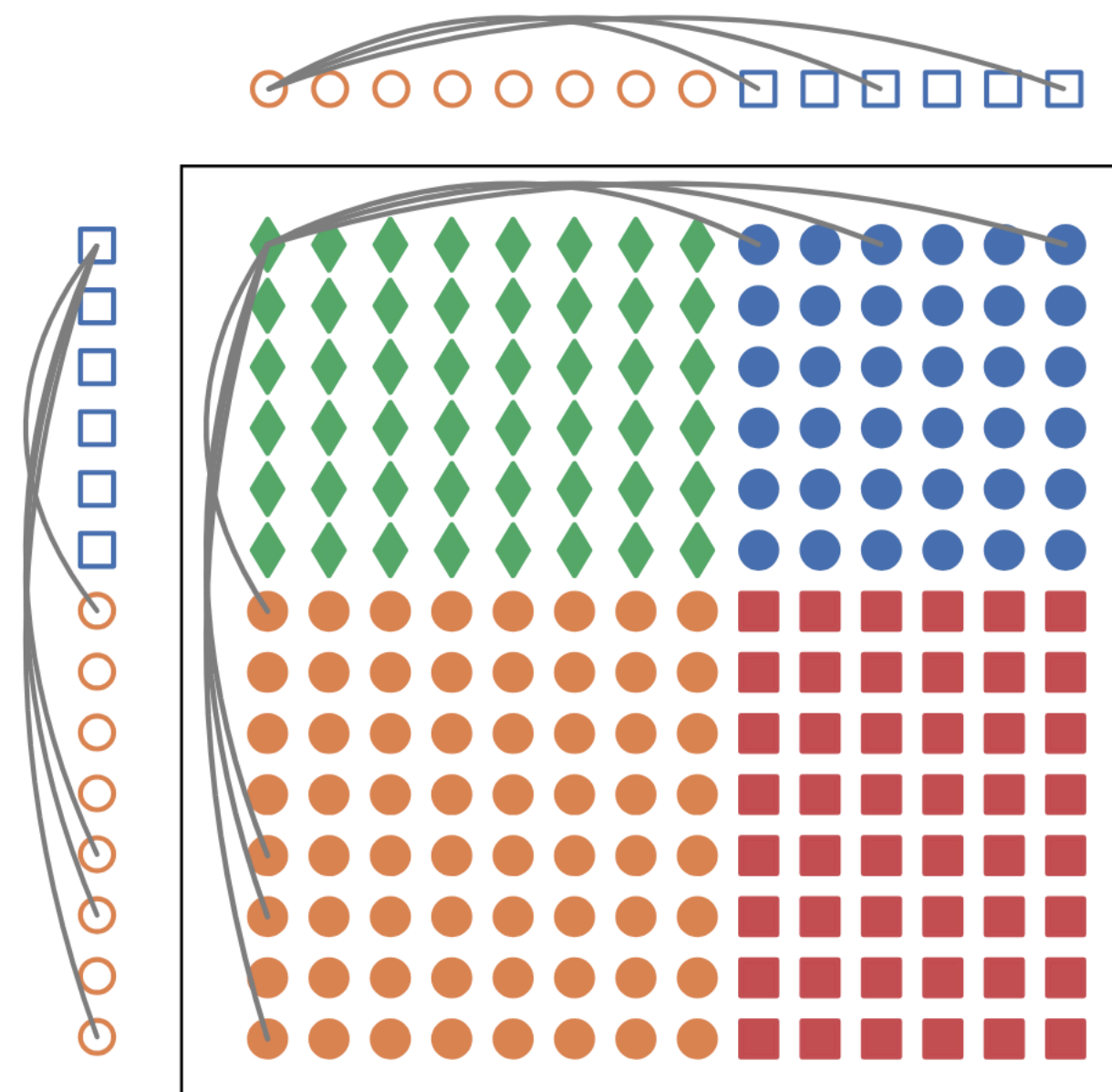


# QLDPC Codes

QLDPC codes enable fault-tolerant memories at low space overhead.



Bivariate Bicycle Code



Hypergraph Product Code

$$H_X = \begin{pmatrix}
 F_0 & F_1 & F_2 & F_3 & F_4 & F_5 & G_0 & G_1 & G_2 & G_3 & G_4 & G_5 \\
 F_5 & F_0 & F_1 & F_2 & F_3 & F_4 & G_5 & G_0 & G_1 & G_2 & G_3 & G_4 \\
 F_4 & F_5 & F_0 & F_1 & F_2 & F_3 & G_4 & G_5 & G_0 & G_1 & G_2 & G_3
 \end{pmatrix}$$

$$H_Z = \begin{pmatrix}
 G_0^T & G_5^T & G_4^T & G_3^T & G_2^T & G_1^T & F_0^T & F_5^T & F_4^T & F_3^T & F_2^T & F_1^T \\
 G_1^T & G_0^T & G_5^T & G_4^T & G_3^T & G_2^T & F_1^T & F_0^T & F_5^T & F_4^T & F_3^T & F_2^T \\
 G_2^T & G_1^T & G_0^T & G_5^T & G_4^T & G_3^T & F_2^T & F_1^T & F_0^T & F_5^T & F_4^T & F_3^T
 \end{pmatrix}$$

Affine Permutation Matrices (APM) Code

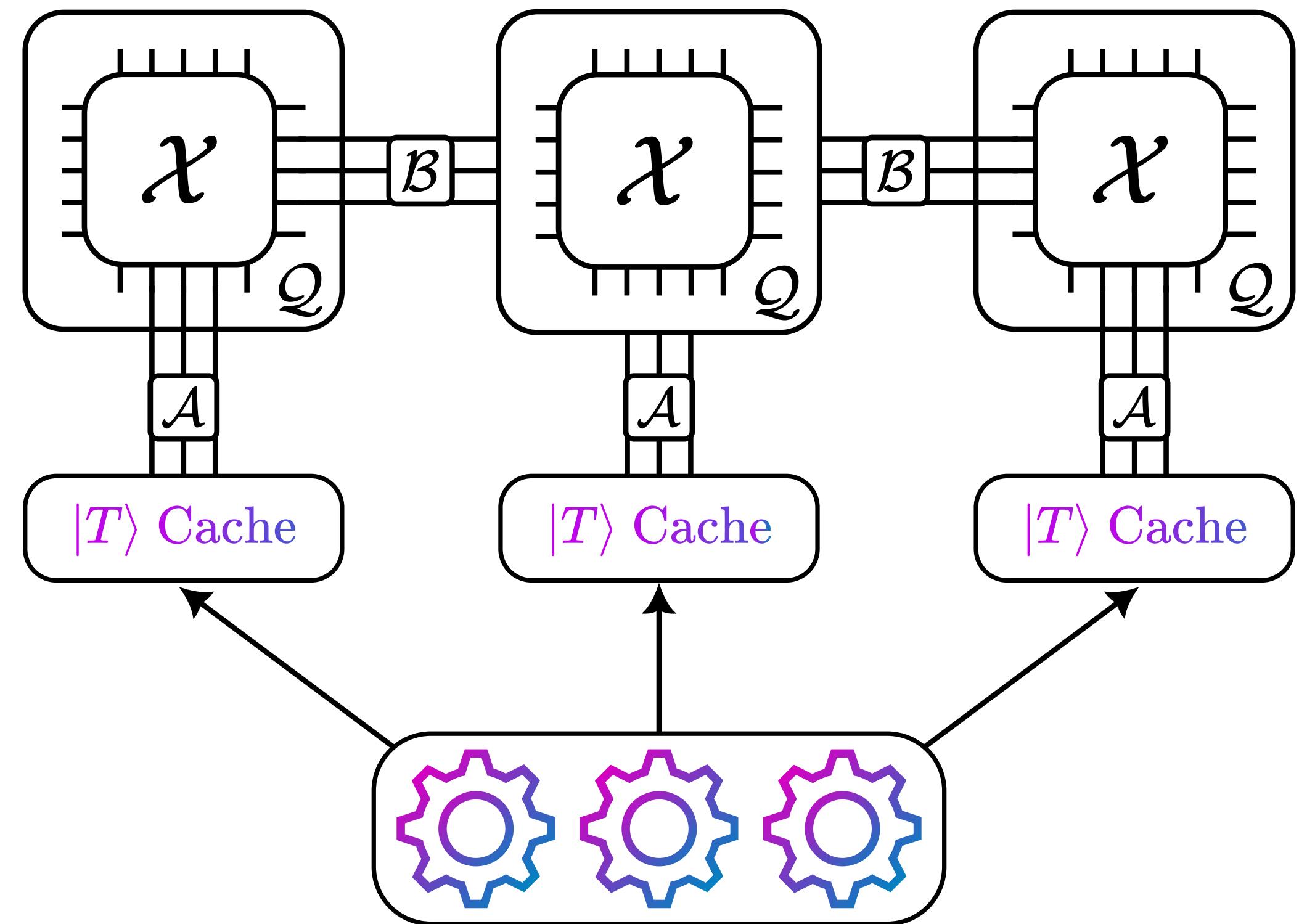
Lots of work in the past two years that study logical computation on QLDPC codes.

# Code Surgery for QLDPC Codes

Surgery is a method to perform logical measurement on QLDPC codes using ancilla systems.

Many desirable properties:

- Provably fault-tolerant and low overhead.<sup>[1,2]</sup>
- **Good finite-size performance.**<sup>[3 - 10]</sup>
- Highly versatile:
  - Can build an extractor system to measure arbitrary logicals sequentially.<sup>[11]</sup>
  - Or in parallel with different designs.<sup>[3,12,13]</sup>
- **Can be adapted to many hardware platforms.**<sup>[5,6,8,10,11]</sup>



Extractor Architectures,  
a model for surgery architectures

# The Need for Speed

Aforementioned surgery systems all take  $O(d)$  time to implement one round of gates.

◆ This is a crucial bottleneck. After all, we cannot build more time.

**This talk: breaking the  $O(d)$  time barrier to implement surgery in  $O(1)$  time.**

## **Formulation of fast surgery as constant-time surgery [1]**

◆  $O(d)$  surgery operations in  $O(d)$  time.

## **Construction on 2-dimensional hypergraph product codes [2]**

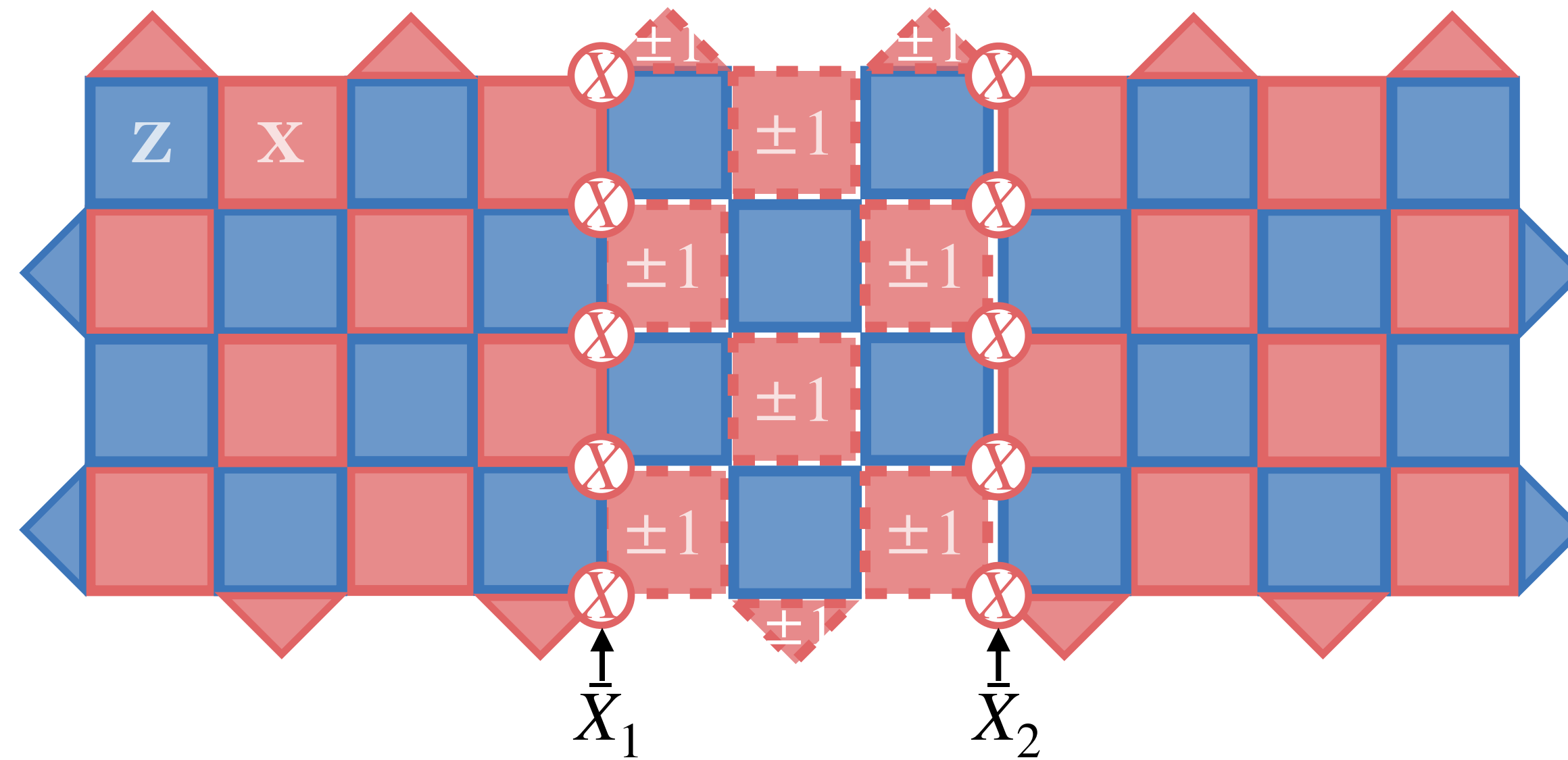
◆ Fast, partially addressable and parallel measurements in almost constant space overhead, on any hypergraph product code.

**Next: why does surgery take  $O(d)$  time?**

# Lattice surgery on the surface code

*Ex: Measuring  $\bar{X}_1\bar{X}_2$*

$$\bar{X}_1\bar{X}_2 = \prod \pm 1$$



- Switch to deformed/merged code where logical operator to be measured = products of stabilizers<sup>[1]</sup>
- Measurement errors  $\rightarrow$  need to repeat for  $O(d)$  rounds.

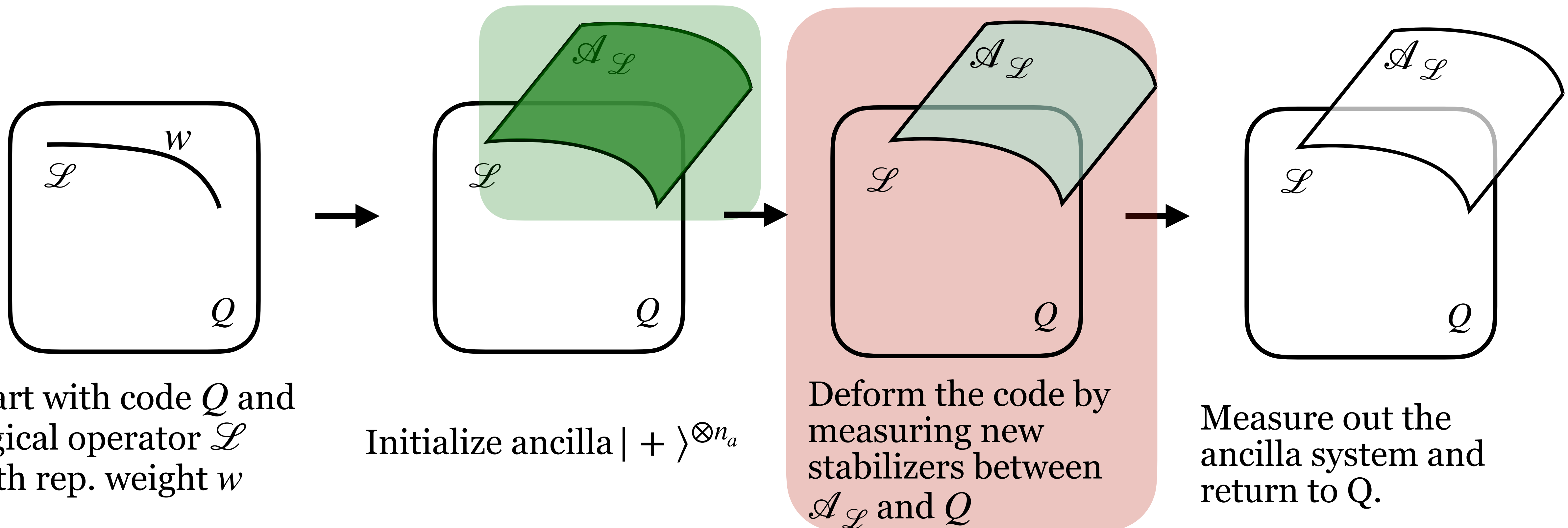
[1] Vuillot *et al.* NJP **21** (2019).

# Generalized Code Surgery for qLDPC codes

- Code surgery generalizes lattice surgery to logical measurement on qLDPC codes<sup>[1-6]</sup>.
- **Low space overhead, large time overhead**

$n_a = O(w \log(w))$ <sup>[7]</sup> space overhead

Repeat for  $O(d)$  rounds.



# Generalized Code Surgery for qLDPC codes

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Repeat for  $O(d)$  rounds.



Start with code  $Q$  and logical operator  $\mathcal{L}$  with rep. weight  $w$

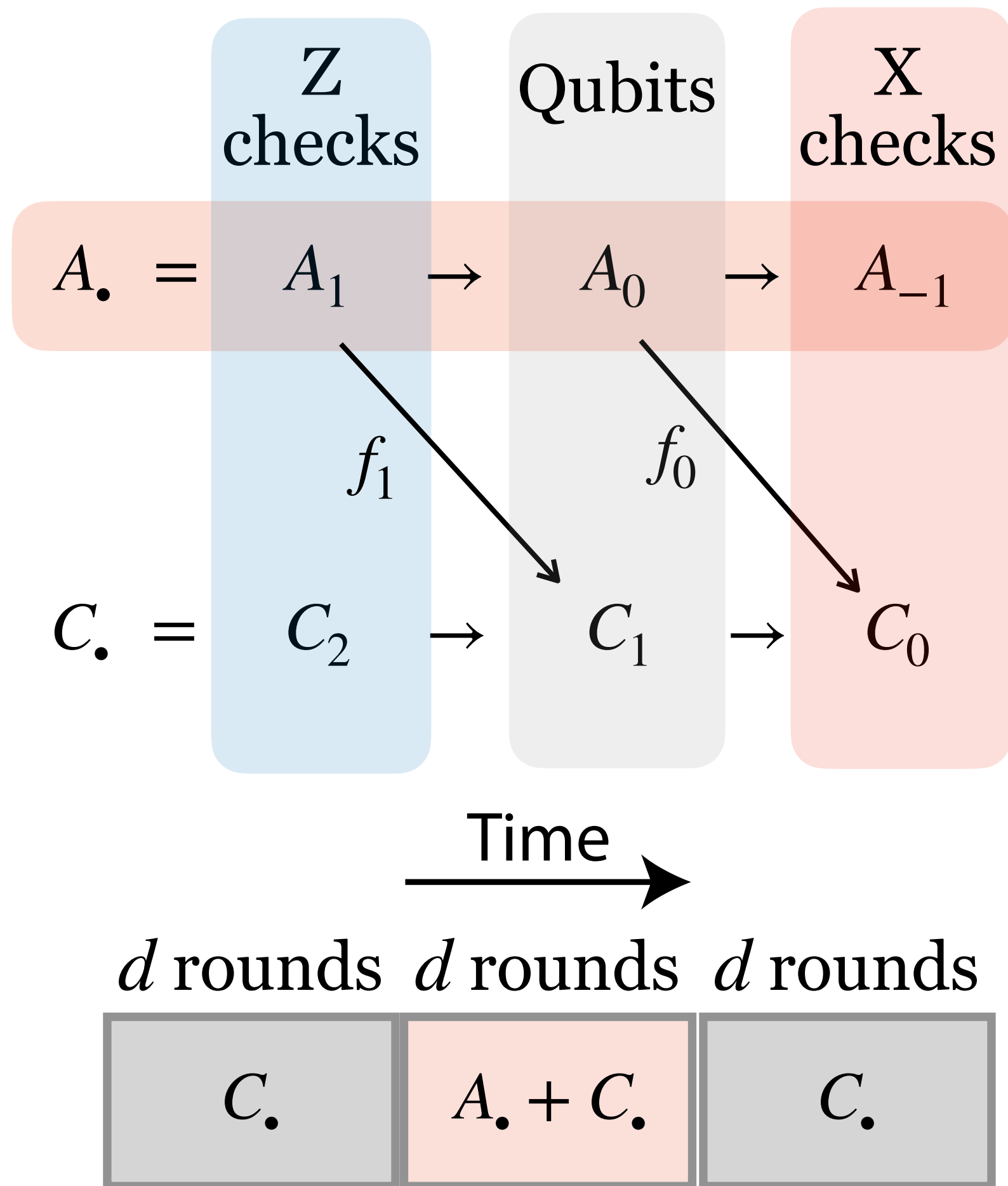
Initialize ancilla  $|+\rangle^{\otimes n_a}$

Deform the code by measuring new stabilizers between  $A_{\mathcal{L}}$  and  $Q$

Measure out the ancilla system and return to  $Q$ .

# Regular surgery

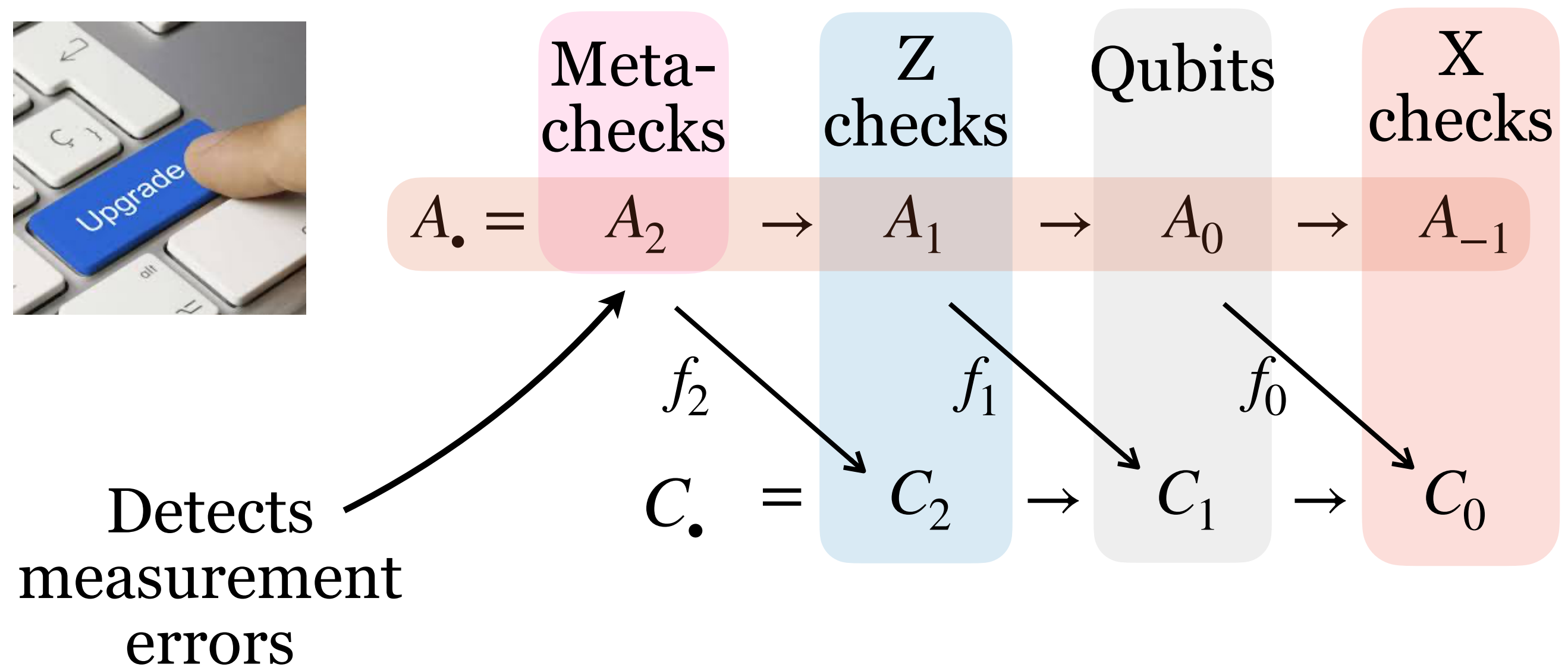
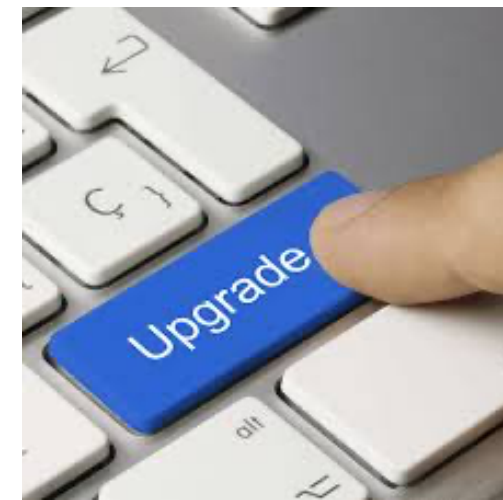
3-term chain complex for **regular code surgery, Z measurement**



# → Fast surgery

4-term chain complex for **fast surgery**

Take a **4-term** chain ancilla complex,  $A_•, f_• : A_• \rightarrow C_•$ , st:



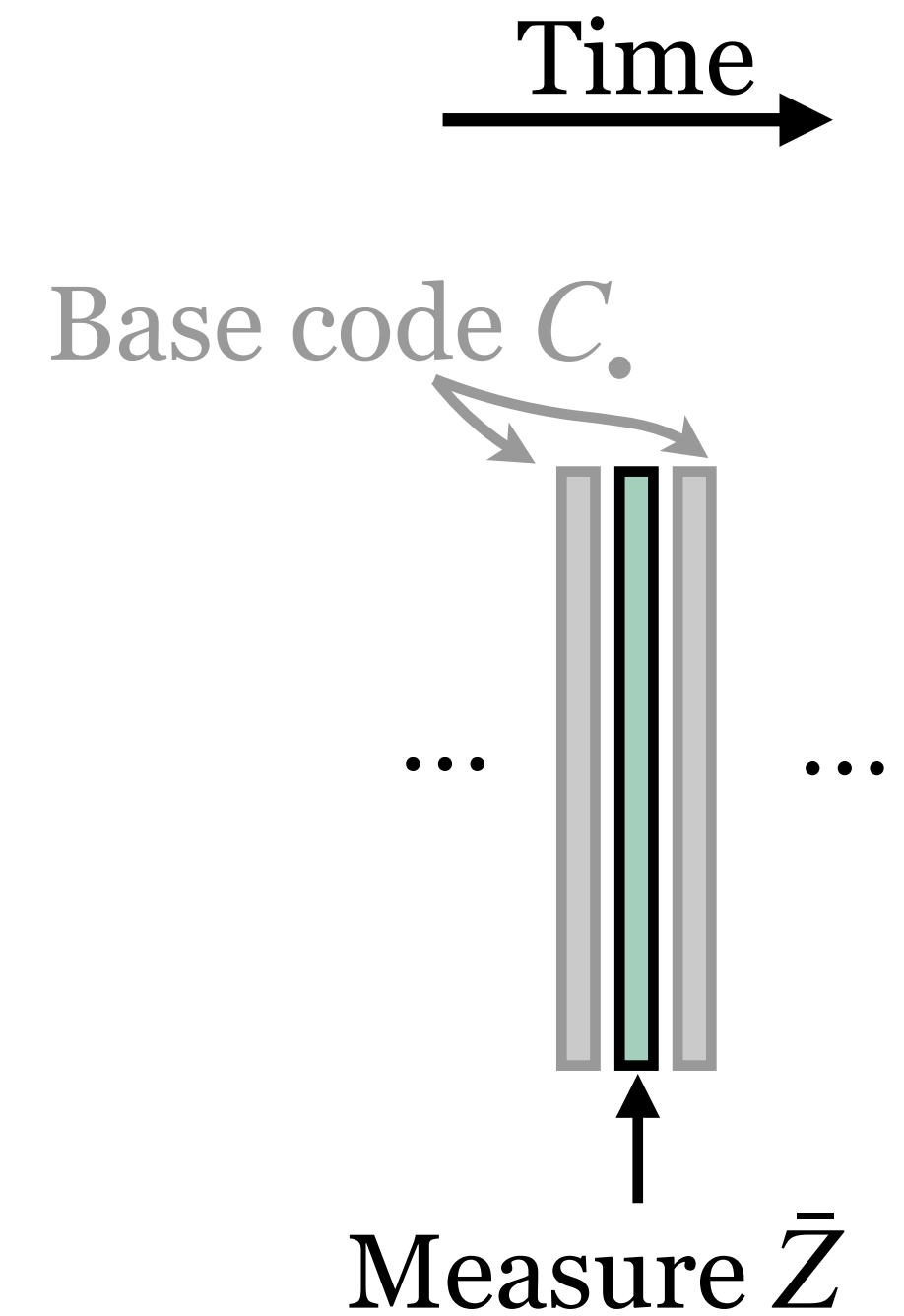
**What is the surgery procedure now?**

*Want fault-distance  $d$  for data qubit & measurement errors*

# Is this fault-tolerant?

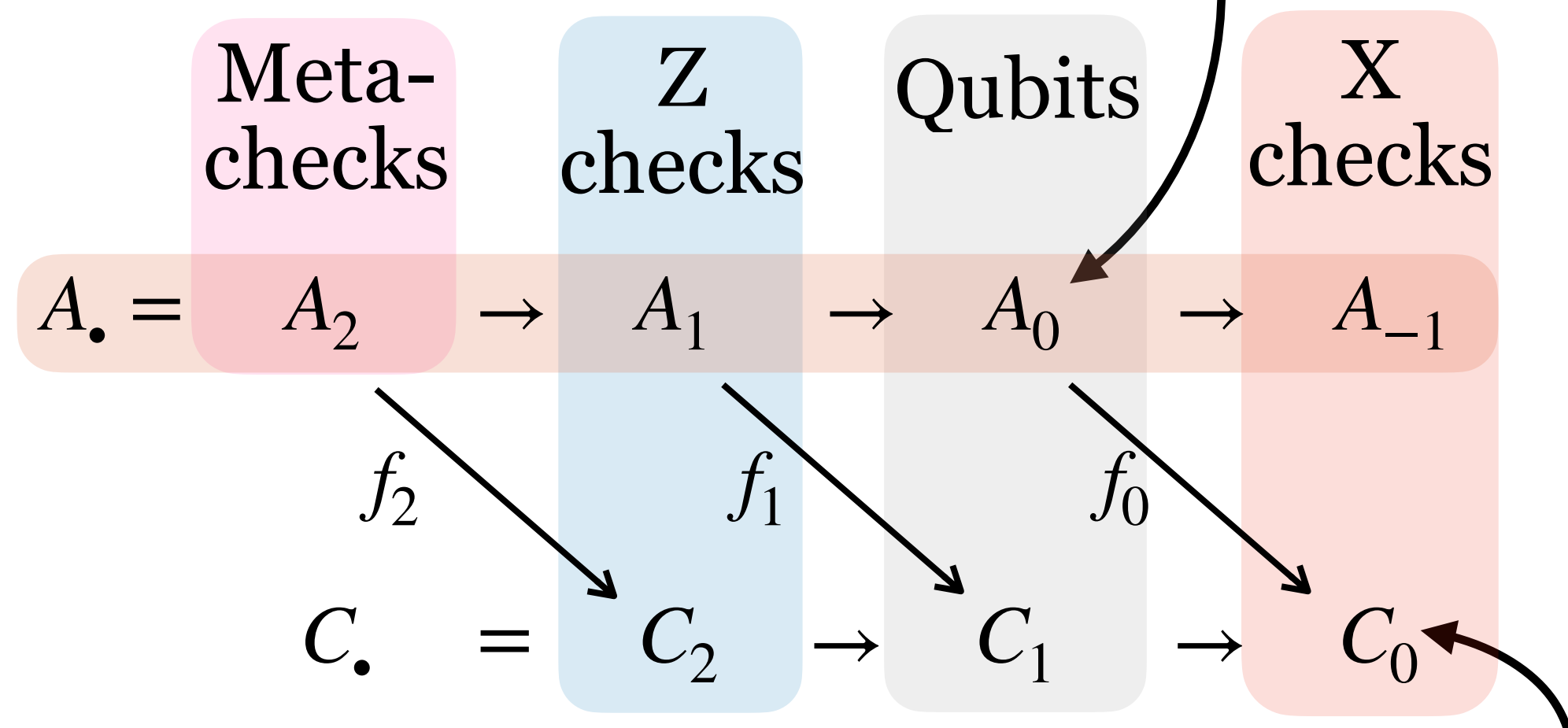


1. 1 round in the base code  $C_\bullet$ .
2. 1 round in the deformed code (measure  $\bar{Z}$ )
3. 1 round in the base code  $C_\bullet$ .



Start: init.  $A_0$  qubits in  $|+\rangle$   
 Finish: measure  $A_0$  qubits in  $X$

**Measure  $\bar{Z}$**



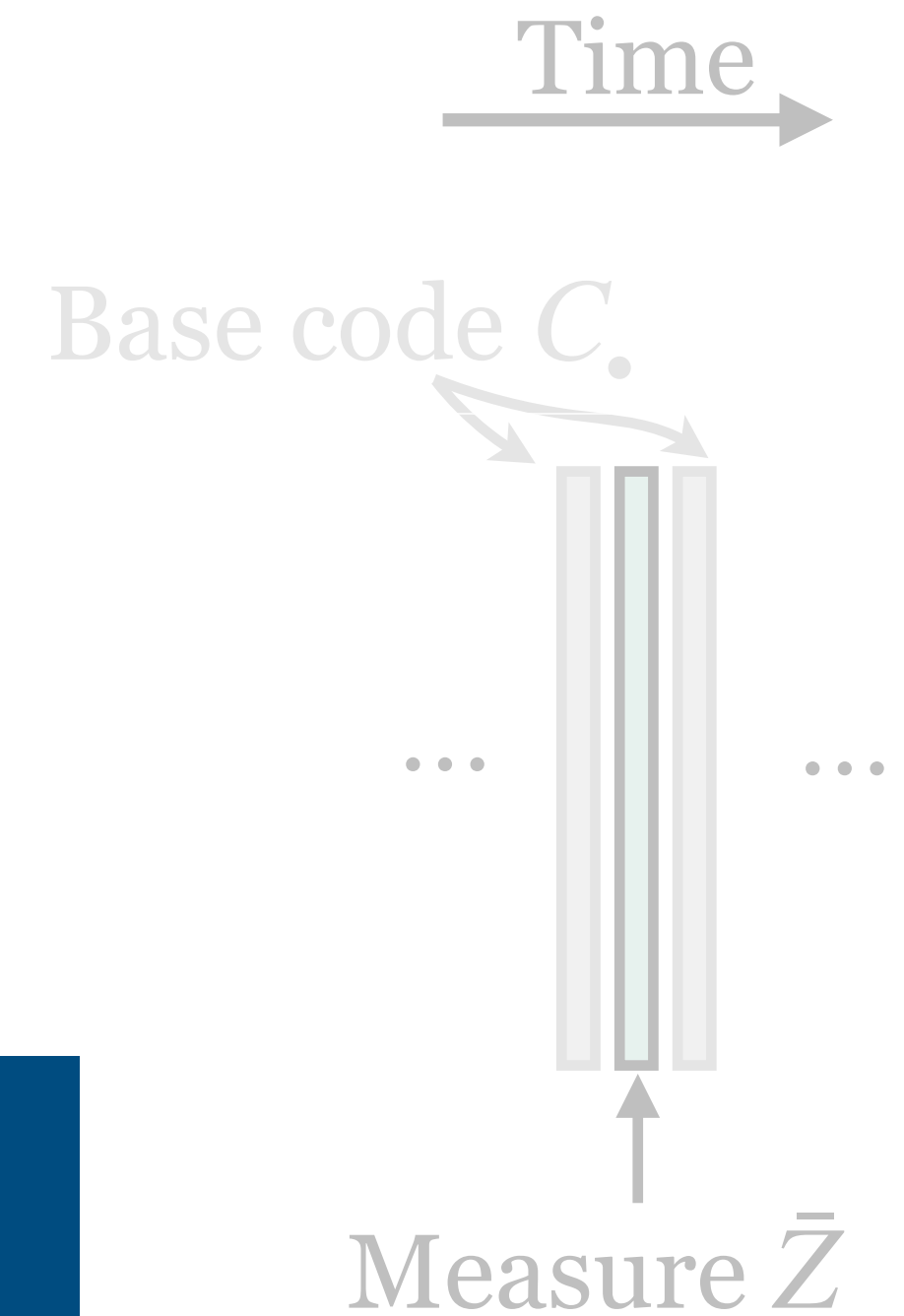
Updated after  $X$  measurement, *random*.

- State preparation-like error. Pauli frame is random after surgery.
- Only 1 round in  $C_\bullet$  to learn the new stabilizer values  $\rightarrow$  incorrectly inferred Pauli frame
- Can affect future logical operations

# Is this fault-tolerant?



1. 1 round in the base code  $C_\bullet$ .
2. 1 round in the deformed code (measure  $\bar{Z}$ )
3. 1 round in the base code  $C_\bullet$ .

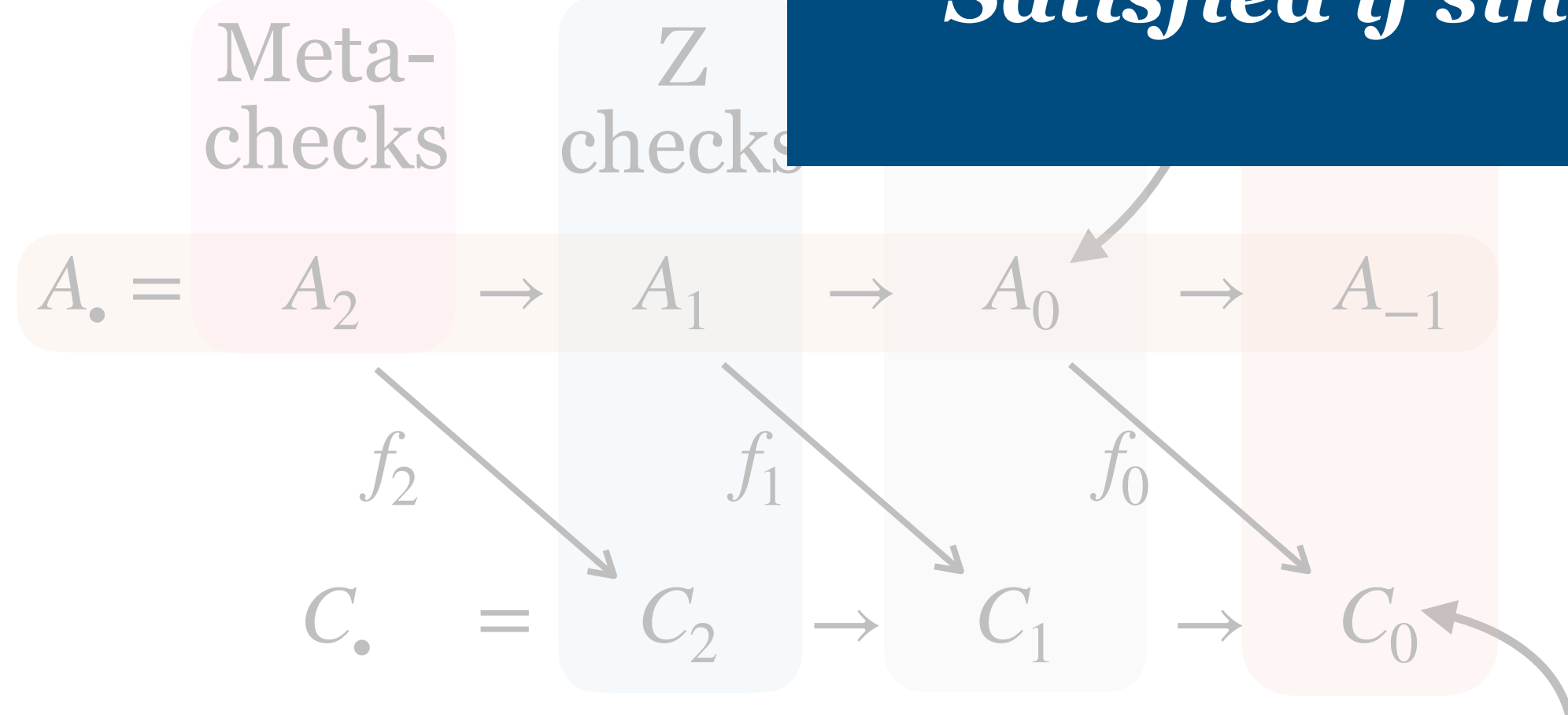


Start: init.  $A_0$  qubits  
 Finish: measure  $A_0$  qubits

**Base code  $C_\bullet$  must infer syndromes w/  
 1 round of syndrome extraction**

**Satisfied if single-shot state preparation**

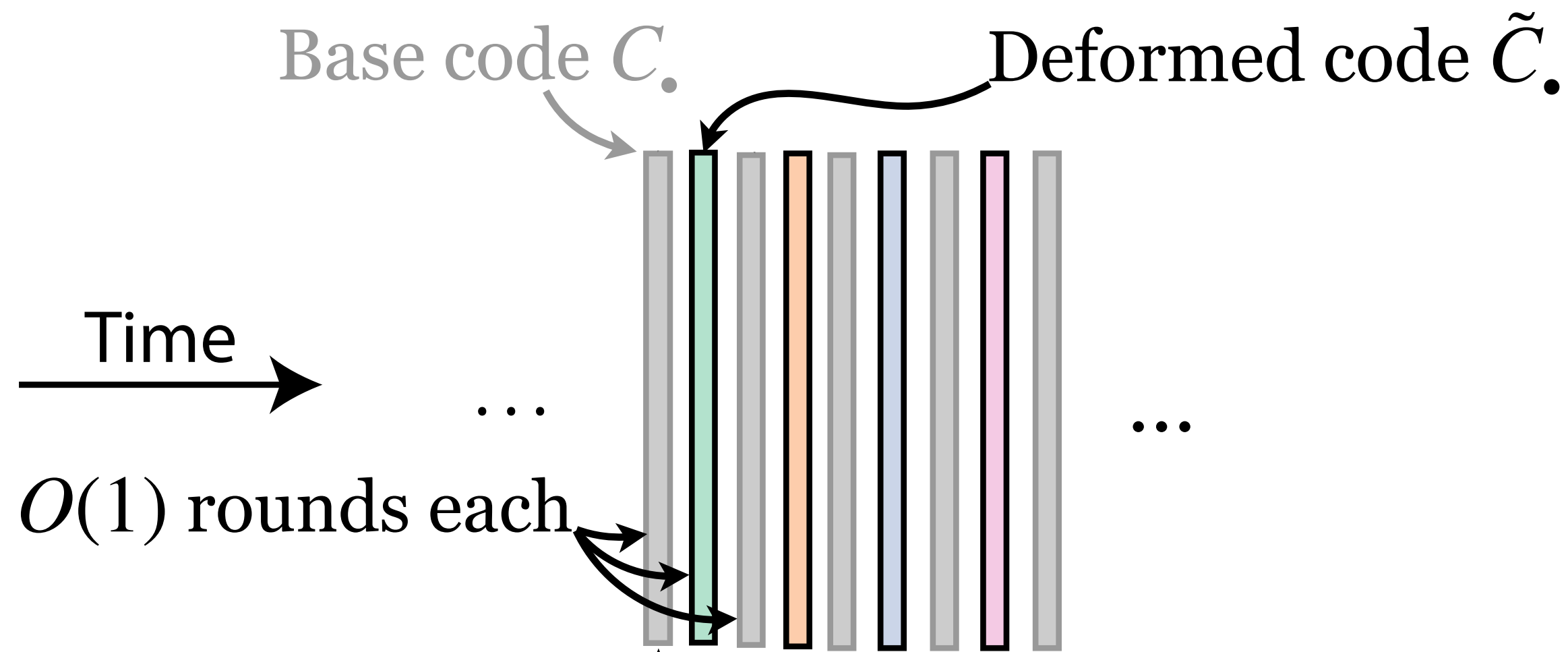
Measure  $\bar{Z}$



Updated after X measurement, *random*.

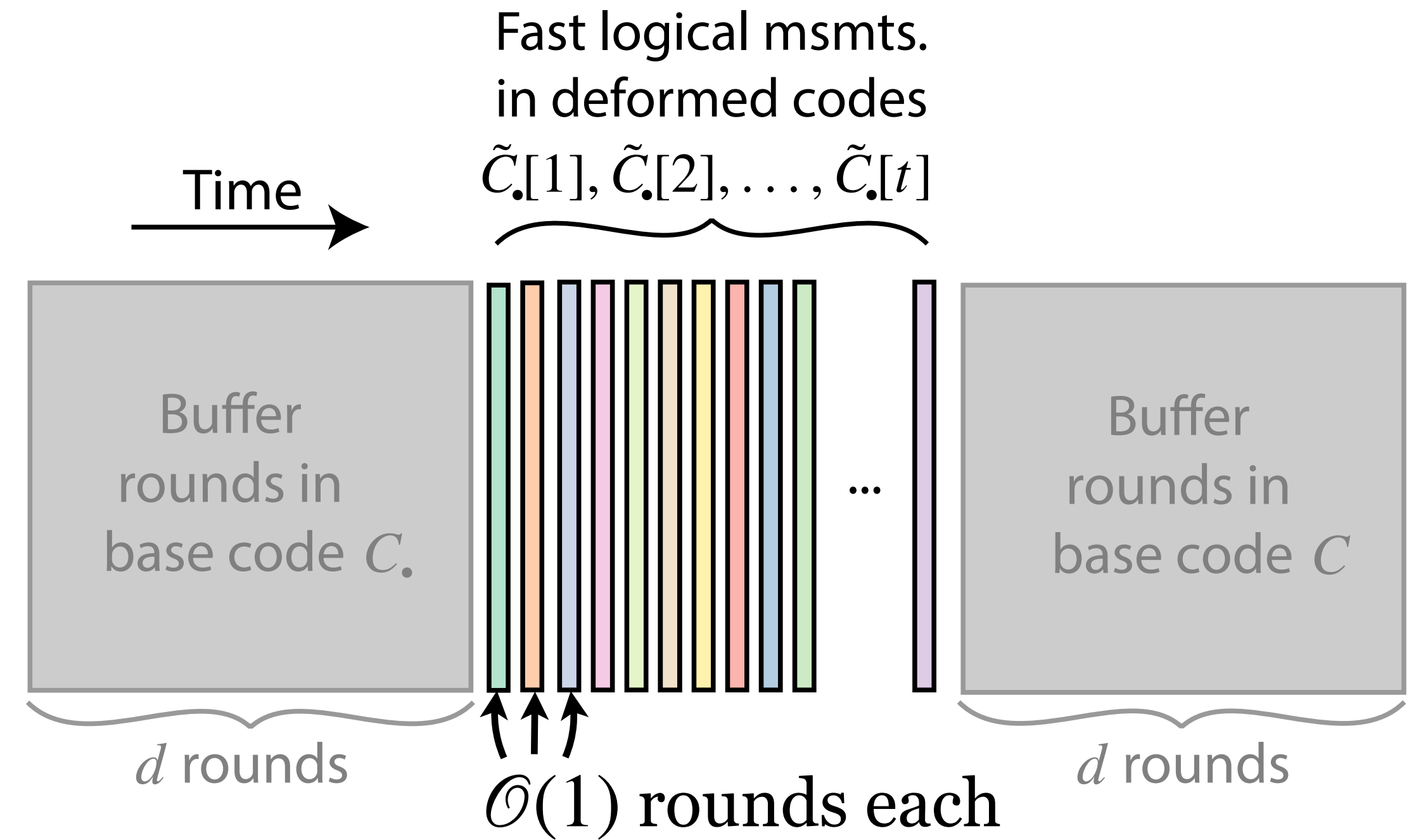
- Only 1 round in  $C_\bullet$  to learn the new stabilizer values  $\rightarrow$  incorrectly inferred Pauli frame
- Can affect future logical operations

## Single-shot surgery [1-3]



- Requires **single-shot preparation of deformed code**, and **base code  $C$** .
- **Restricts what  $C$  can be.**
  - **Base code must have meta-checks**
- Concurrent work<sup>[3]</sup>: deformed  $\tilde{C}$  may introduce large space overhead:  $\tilde{O}(nkd)$

## Constant-time surgery [4, 5]



- $C$  need not be single-shot state preparable
- Sufficient detectors to infer the Pauli frame after each msmt
- Each  $\tilde{C}[i]$  needs to satisfy certain properties to ensure fault-tolerance

[1] Hillman *et al.* [2410.12963]

[2] Tan, Hong, Lin *et al.* [2510.08552]

[3] Baspin *et al.* [2510.04521]

[4] Cowtan, He, Williamson, Yoder. [2510.14895]

[5] Chang, He, Yoder, Zhu, Jochym-O'Connor. [2603.02157]

# Result 1: Fault-tolerance conditions for constant time surgery

Let  $C_\bullet$  be a CSS code of distance  $d$ , and consider a **sequence of surgery operations** defined by ancilla complexes  $A[1]_\bullet, \dots, A[t]_\bullet$  and chain maps  $f[1] : A[1]_\bullet \rightarrow C_\bullet, \dots, f[t] : A[t]_\bullet \rightarrow C_\bullet$ .

Then the constant-time surgery protocol has **fault distance  $d$**  if:

- ◆ Each ancilla complex  $A[i]_\bullet$  has meta-check distance at least  $d$ .

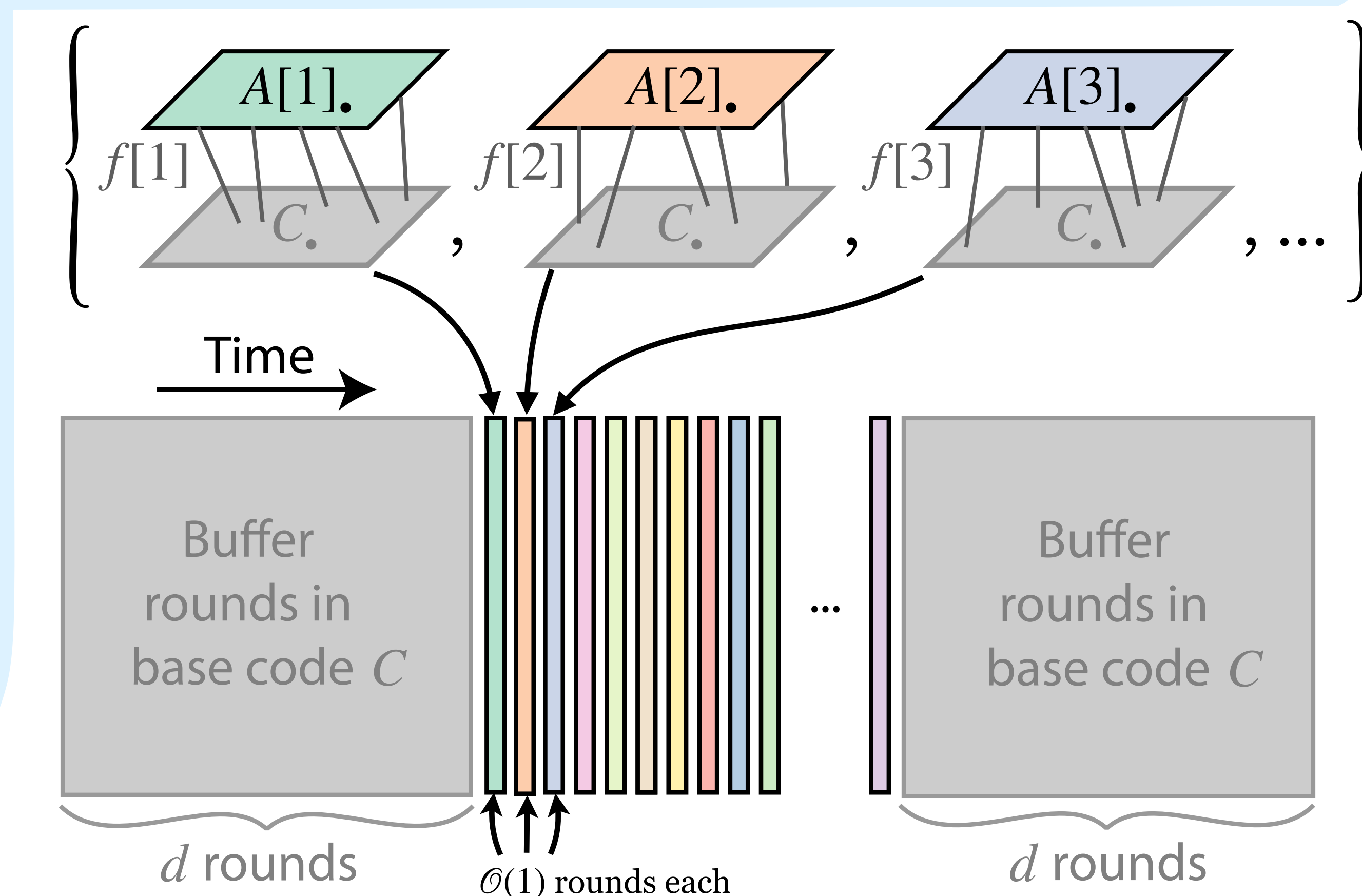
*Can protect up to  $d$  ancilla measurement errors*

- ◆ The compacted code  $CC_\bullet^*$  has distance  $\geq d$ .

*Unmeasured logical qubits have high distance*

- ◆ Chain maps are partially injective.

*Error propagation is bounded*



\*a CSS code defined by  $t$  surgery operations

## Result 2: Explicit low-overhead construction for the HGP code

Given an  $[[n, k, d]]$  hypergraph product code, we can construct a sequence of **LDPC** ancillary complexes that:

- ◆ **Large fault distance:** Satisfies fault tolerance conditions for constant time surgery.
- ◆ **Parallel, partially addressable measurements:** Parallelized measurement of rows (and columns) of  $Z$  ( $X$ ) logical operators.
- ◆ **Constant time per measurement:** Each surgery operation takes  $\mathcal{O}(1)$  **time** in amortization.
- ◆ **Low space overhead:** Each complex **uses**  $\mathcal{O}(n \log(n))$  **physical ancilla qubits**.

# 2D hypergraph product codes

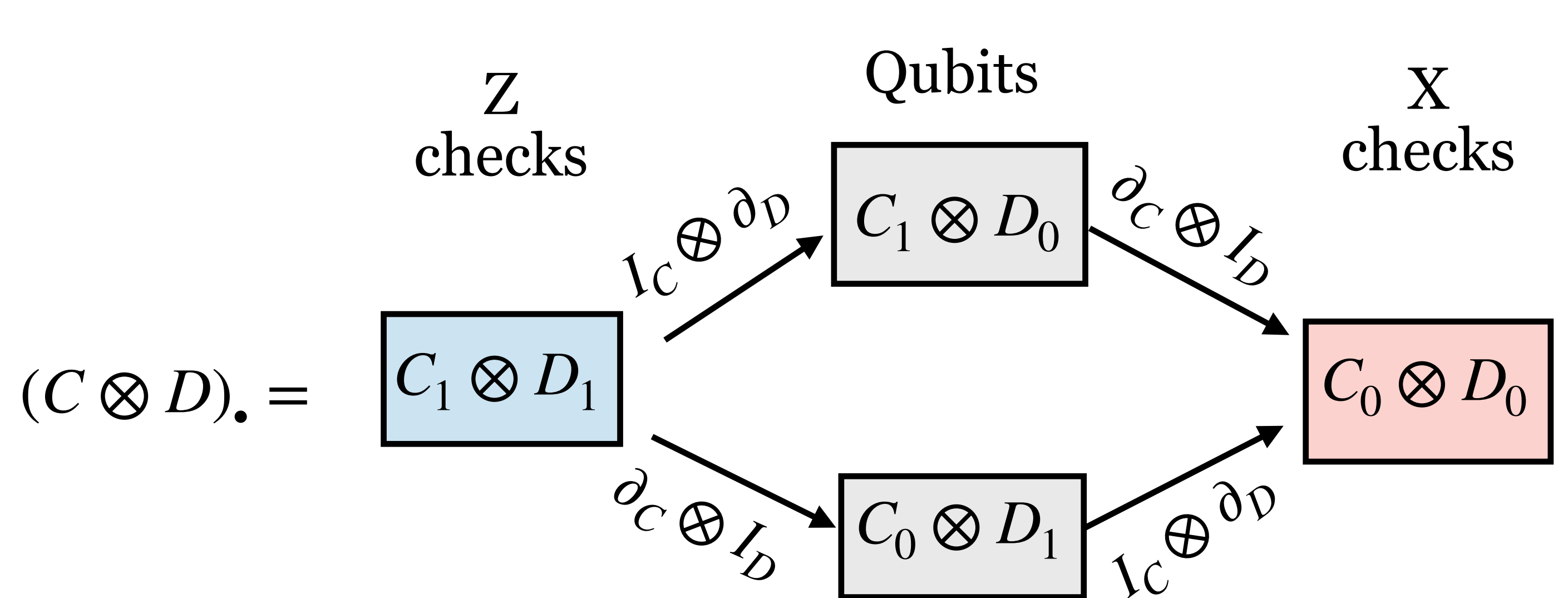
Family of codes that can achieve constant encoding rate and sublinear distance  $[[n, O(n), O(\sqrt{n})]]$

“2D” refers to the product of *two* classical codes

Classical code 1:  $C_\bullet = C_1 \xrightarrow{\partial_C} C_0$   
Bits Checks

Classical code 2:  $D_\bullet = D_1 \xrightarrow{\partial_D} D_0$

The hypergraph product of these codes is



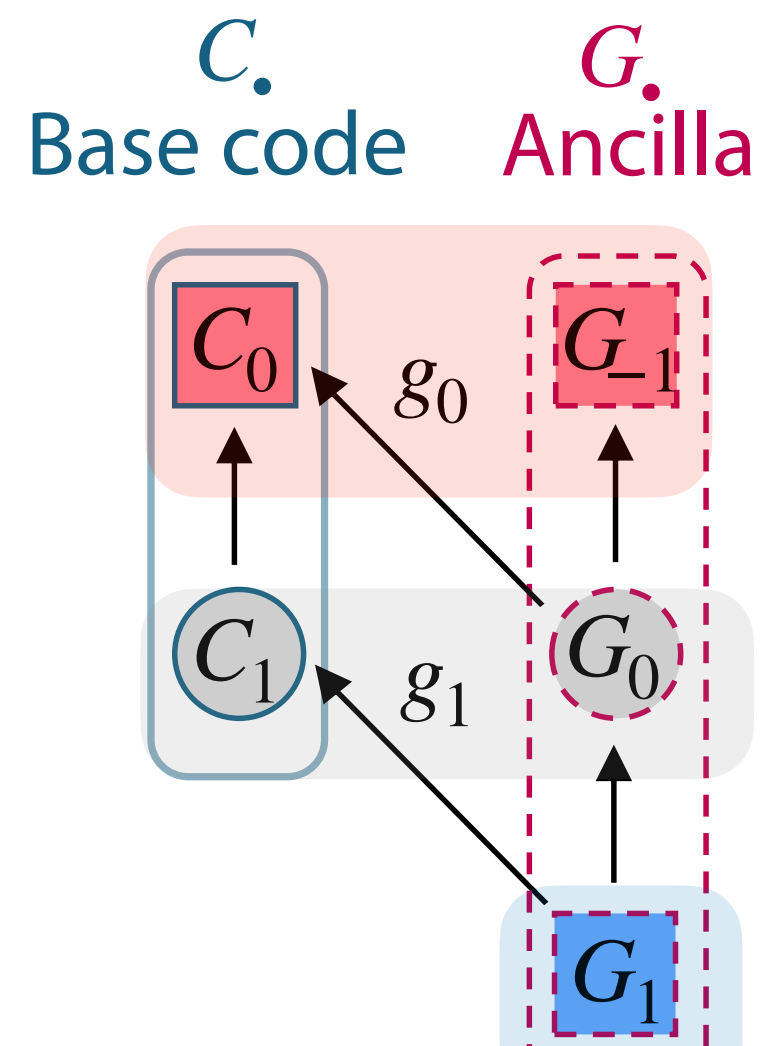
# Constructing constant-time surgery gadgets

1. Apply surgery gadget on the classical code  $C$ .  
The ancilla =  $G$ .

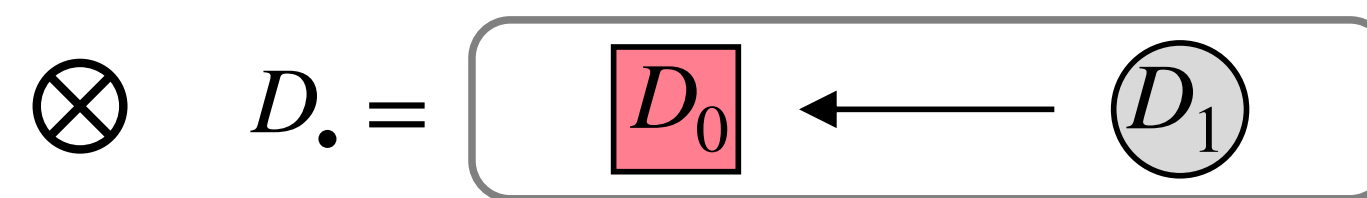
2. Take the tensor product with  $D$ .

3. Arrive at deformed code, “ $(G + C) \otimes D$ ”

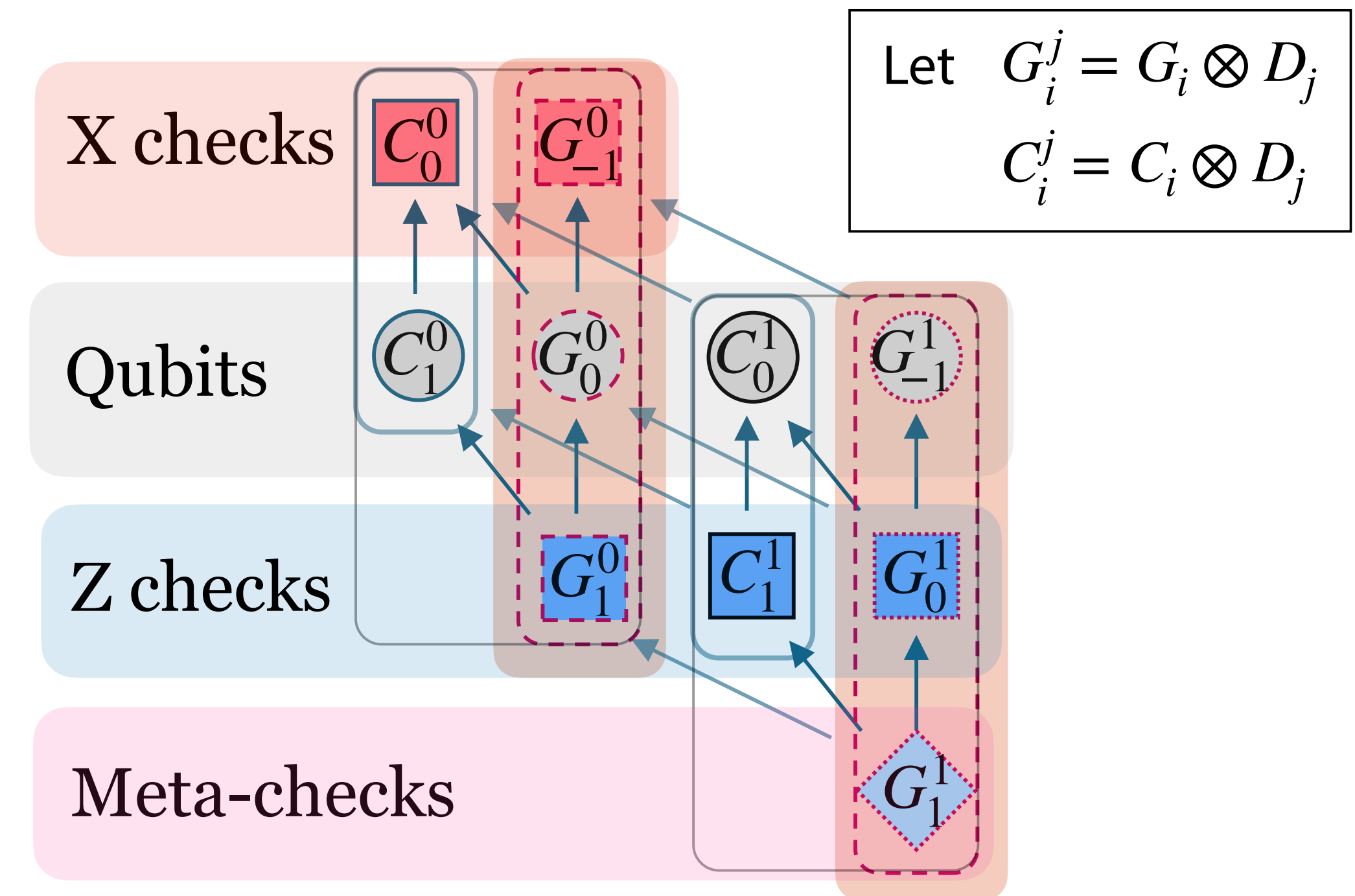
Surgery on classical code  
(3-term chain complex)



Classical code  
(2-term chain complex)



= Quantum code  
(4-term. chain complex)



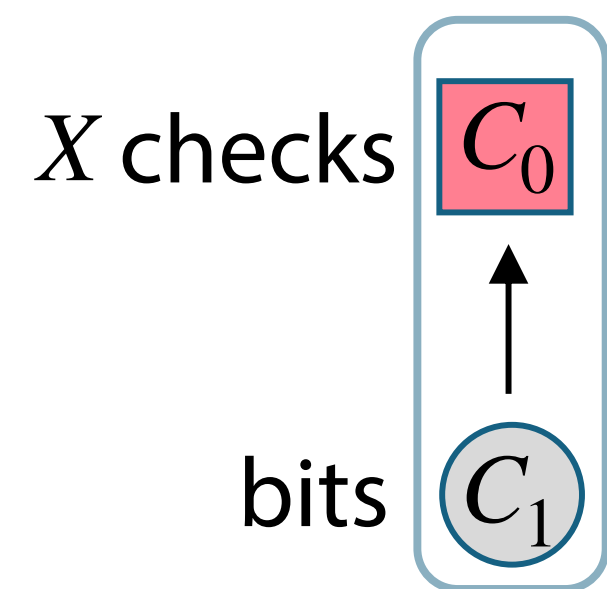
The ancilla complex  $A$  is  $(G \otimes D)$ .

The chain map is  $f: (G \otimes D) \xrightarrow{g \otimes \text{Id}_D} (C \otimes D)$ .

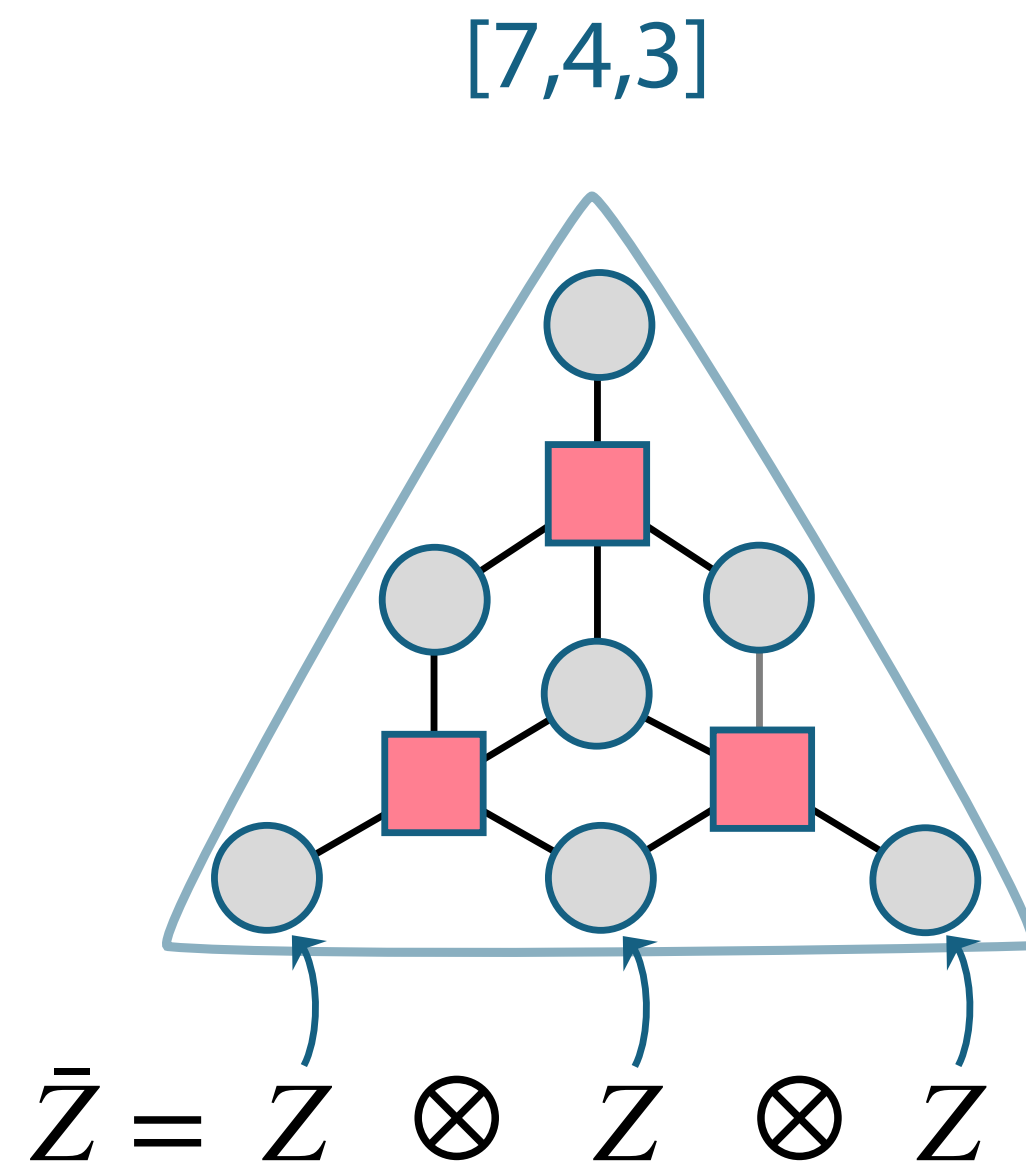
# Ex. Constructing the deformed code

1. Take the base classical code  $C$ .

Chain complex



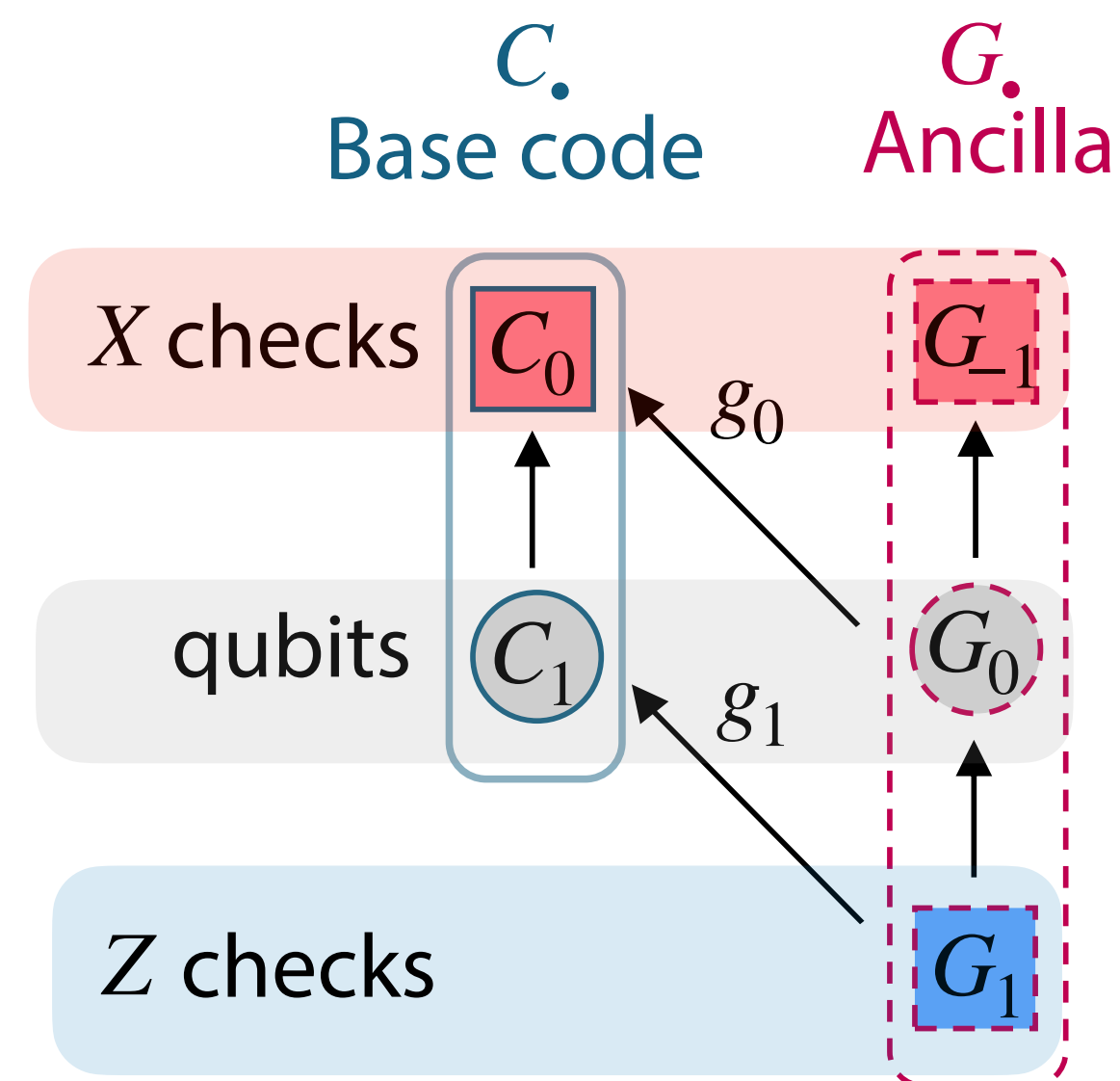
Tanner graph Ex.



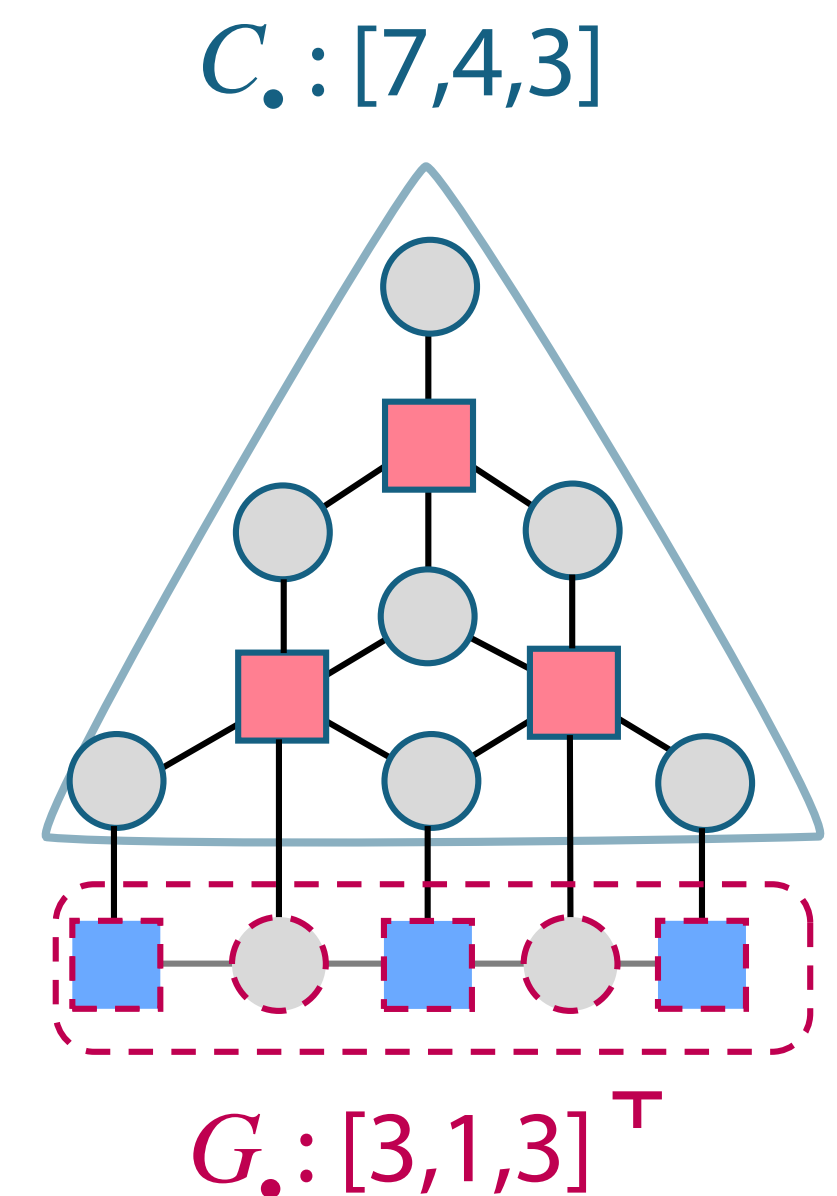
2. Augment the classical code with known surgery techniques<sup>[1,2]</sup>, measure the classical codeword.

$$(g : G_{\bullet} \rightarrow C_{\bullet})$$

Chain complex

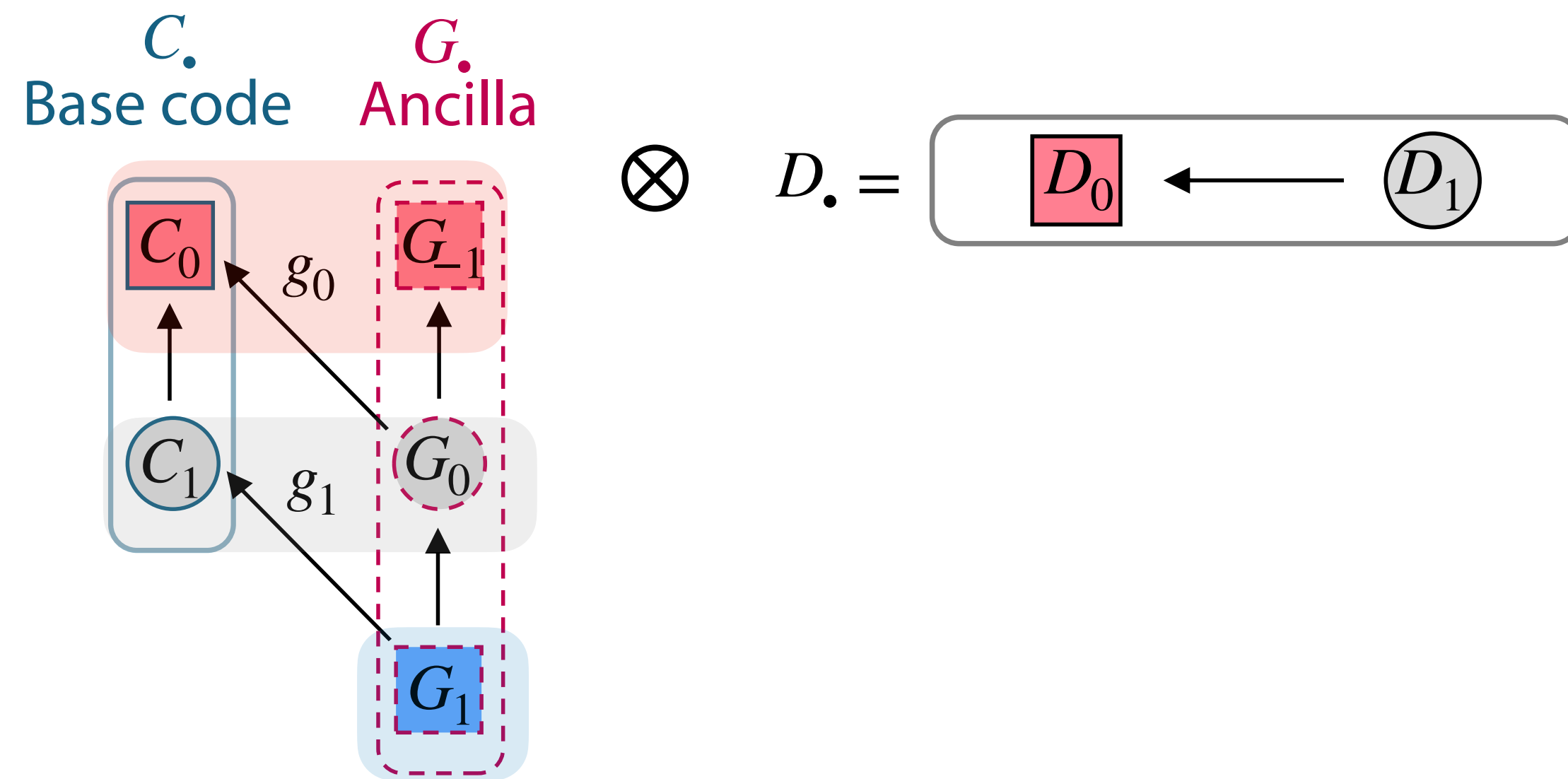


Tanner graph Ex.



# Ex. Constructing the deformed code

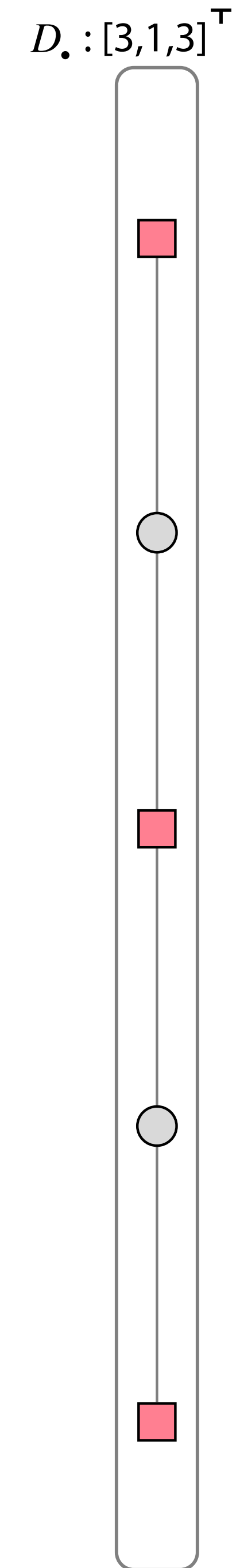
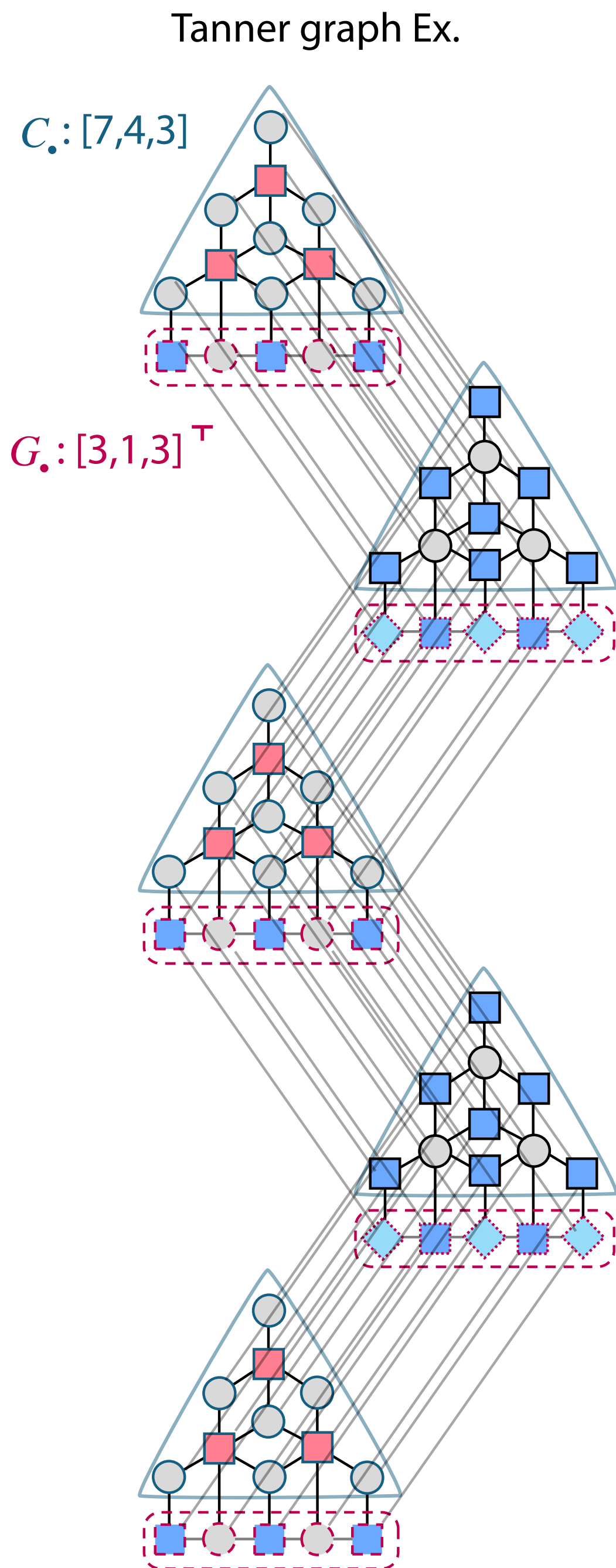
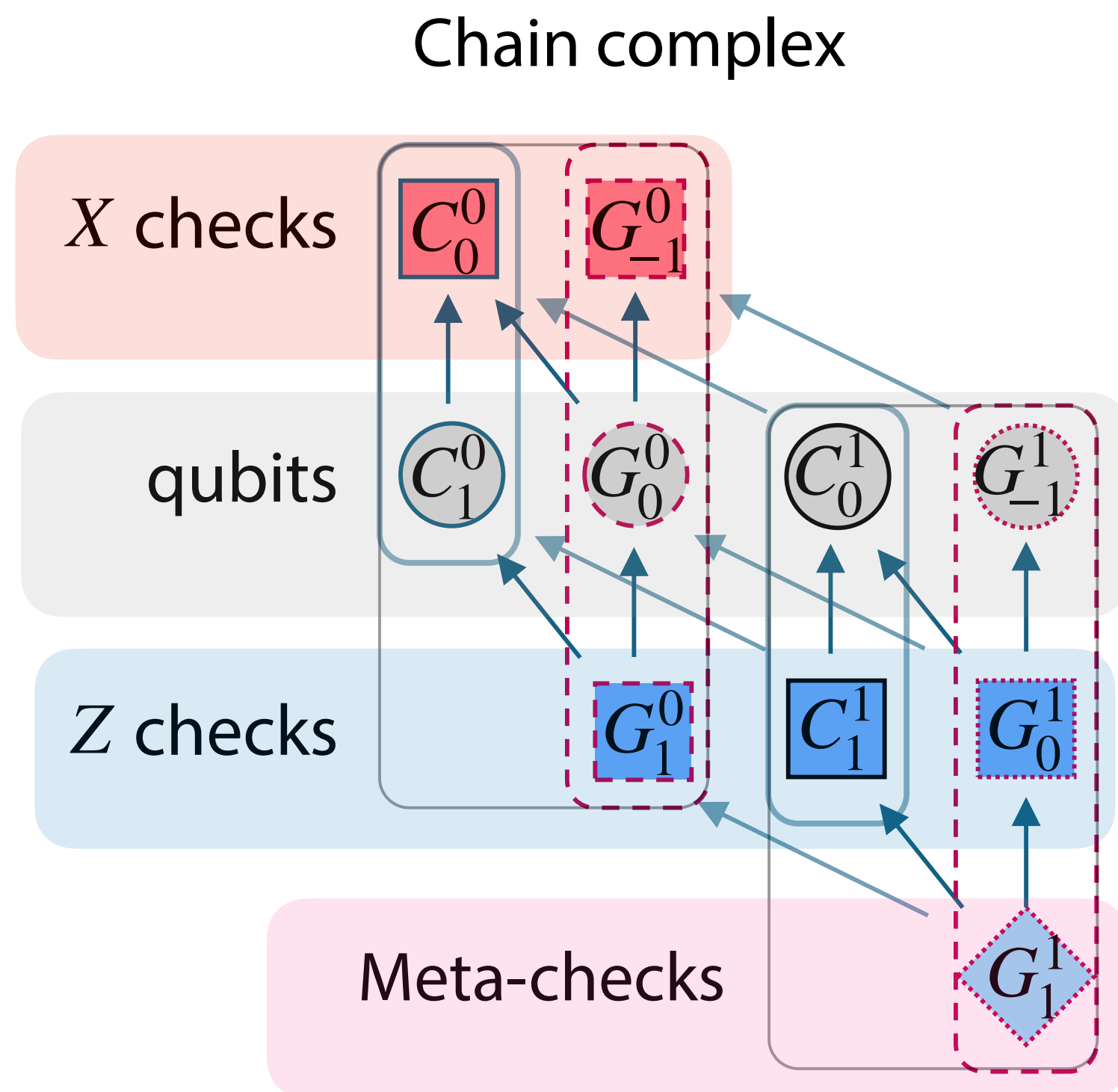
3. Take the hypergraph product with the other base classical code,  $D_\bullet$ .



# Ex. Constructing the deformed code

- Take the hypergraph product with the other base classical code,  $D_\bullet$ .

Let  $G_i^j = G_i \otimes D_j$   
 $C_i^j = C_i \otimes D_j$



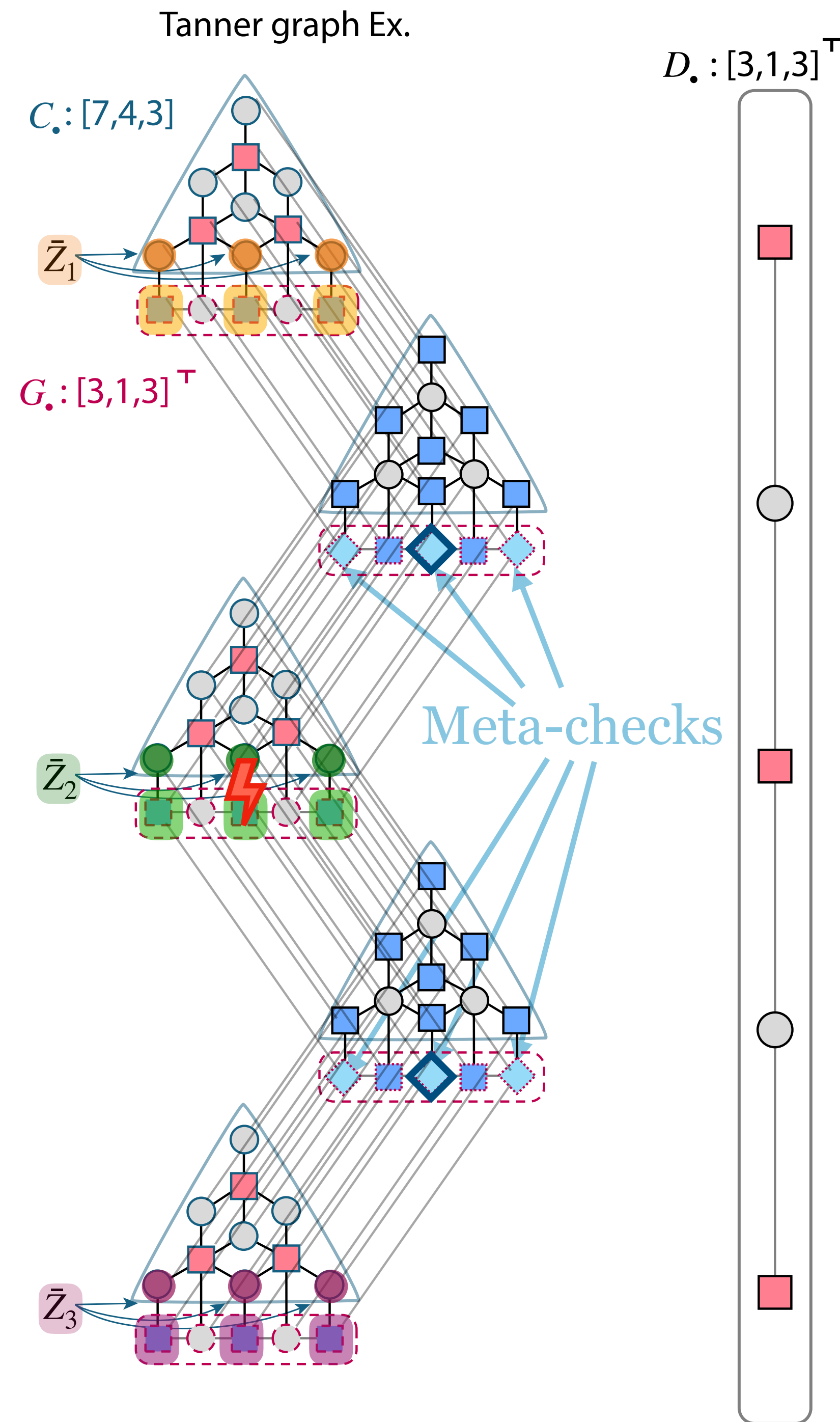
# Ex. Constructing the deformed code

What does measuring the stabilizers of this deformed code accomplish?

- $\bar{Z}_1, \bar{Z}_2, \bar{Z}_3$  are logically equivalent representatives of the same logical operator.
- $\bar{Z}_1, \bar{Z}_2, \bar{Z}_3$  logical representatives are measured simultaneously — each equal a product of ancilla Z checks

**Redundancy** by measuring different *representatives* of the same logical operator.

Ensure fault tolerance against **measurement errors**.



# Summary

- Formulation of constant-time surgery and fault-tolerance conditions
- Given an  $[[n, k, d]]$  hypergraph product code, we can construct a sequence of **LDPC** ancillary complexes that:
  - ☑ **Satisfy *fault-tolerance conditions*** for constant-time surgery
  - ☑ **Some addressibility:** Parallelized measurement of **rows (and columns) of Z (X) logical operators.**
  - ☑ **Constant time:** Each surgery operation takes  $\mathcal{O}(1)$  **time** in amortization.
  - ☑ **Low space overhead:** Each complex **uses  $\mathcal{O}(n \log(n))$  physical ancilla qubits.**

Num. qubits in  $(G \otimes D)$ ?

1. A classical codeword of  $C$ , and each vector space of  $D$ , have size at most  $O(\sqrt{n})$ .
2. Surgery of the classical codeword uses at most  $\tilde{O}(\sqrt{n})^{[1]}$  qubits.
3. Taking the tensor product with  $D$  yields a space overhead  $\tilde{O}(\sqrt{n} \circ \sqrt{n})$

# Discussion and Open directions

- **Entangling measurements** across **two or more hypergraph product codes** that share at least one classical code,  $(C \otimes D)$ ,  $(C' \otimes D)$ ,  $(C'' \otimes D)$ , ... using adapters<sup>[1]</sup>
- Can use extractors on classical code to build **fast, parallel, and partially addressable extractors**
- **Batched computation**<sup>[2]</sup> through parallel measurements on 2D hypergraph product codes
- Practical performance and decoding
- Addressability beyond rows or columns
- Other codes
- Applications & compilation w/ constant-time surgery!

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[1] Swaroop *et al.* [2410.03628]

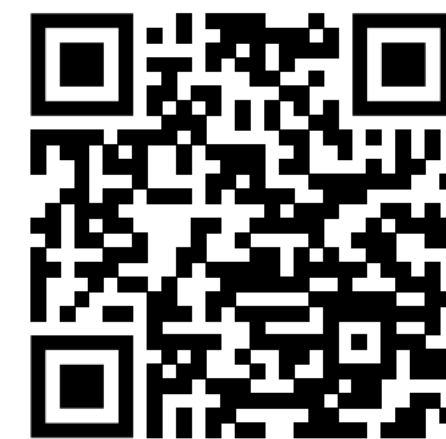
[2] Xu *et al.* [2510.06159]

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Slides here!

