Low-Overhead QLDPC Surgery for Logical Measurements

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The Promise of QLDPC Codes

Quantum error correction is a fundamental building block of large-scale quantum computation. It is an expensive procedure with significant space and time overheads.

- The most studied approach uses the surface code, which has lots of amazing properties: high threshold, 2D connectivity, and more.
- However, surface code costs a lot of physical qubits. To realize 2048-bit factoring, we need ~1000 physical qubits per logical qubit.
- Quantum low-density parity-check (LDPC) codes have high encoding rate, and promise to lower space cost significantly.
- However, performing logical computation on QLDPC codes is a long-standing challenge. Intuitively, as these codes encode information much more compactly, accessing and controlling information become harder.

Our works [1, 2], together with [3] (Poster 39), established code surgery as a promising technique to perform logical computation on QLDPC codes. See also [4], which will be presented on Friday. The methods we developed form the backbone of several subsequent works [5, 6, 7], all of which will be presented on Wednesday.

Logical Measurements and Code Surgery

Code surgery, starting with lattice surgery, is a technique that enables fault-tolerant measurements of logical Pauli operators. Logical measurements enable Pauli-based computation (PBC), which is universal assuming a supply of magic states.

Pauli-based computation: logical measurements + magic state = universal computation.

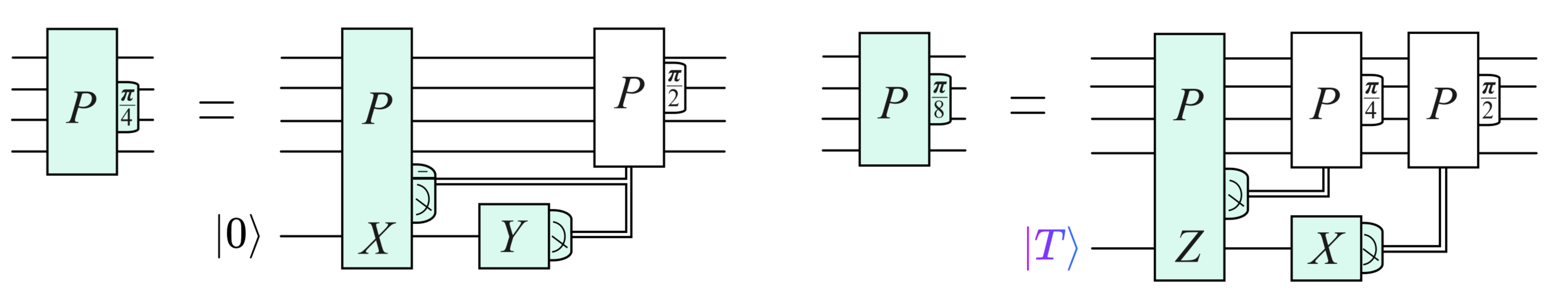


Figure 1: Clifford and non-Clifford gates can be implemented by logical Pauli measurements, supplied with resource states.

Surgery can be formulated in terms of code switching:

- Start with a code $\mathcal{Q}$ with stabilizer group $\mathcal{S}$ and logical operator P;
- Introduce a set of ancilla qubits and define a new stabilizer group $\mathcal{S}'$ such that $P \in \mathcal{S}'$. This new stabilizer code $\mathcal{Q}_P$ is called the measurement code;
- Measure $\mathcal{S}'$ for sufficient number of rounds, which lets us determine the measurement outcome of P;
- Measure $\mathcal{S}$ for sufficient number of rounds, which returns us to $\mathcal{Q}$.

The key idea is that by introducing new stabilizers, the challenging measurement of high-weight P is decomposed into many simple measurements of low-weight stabilizers.

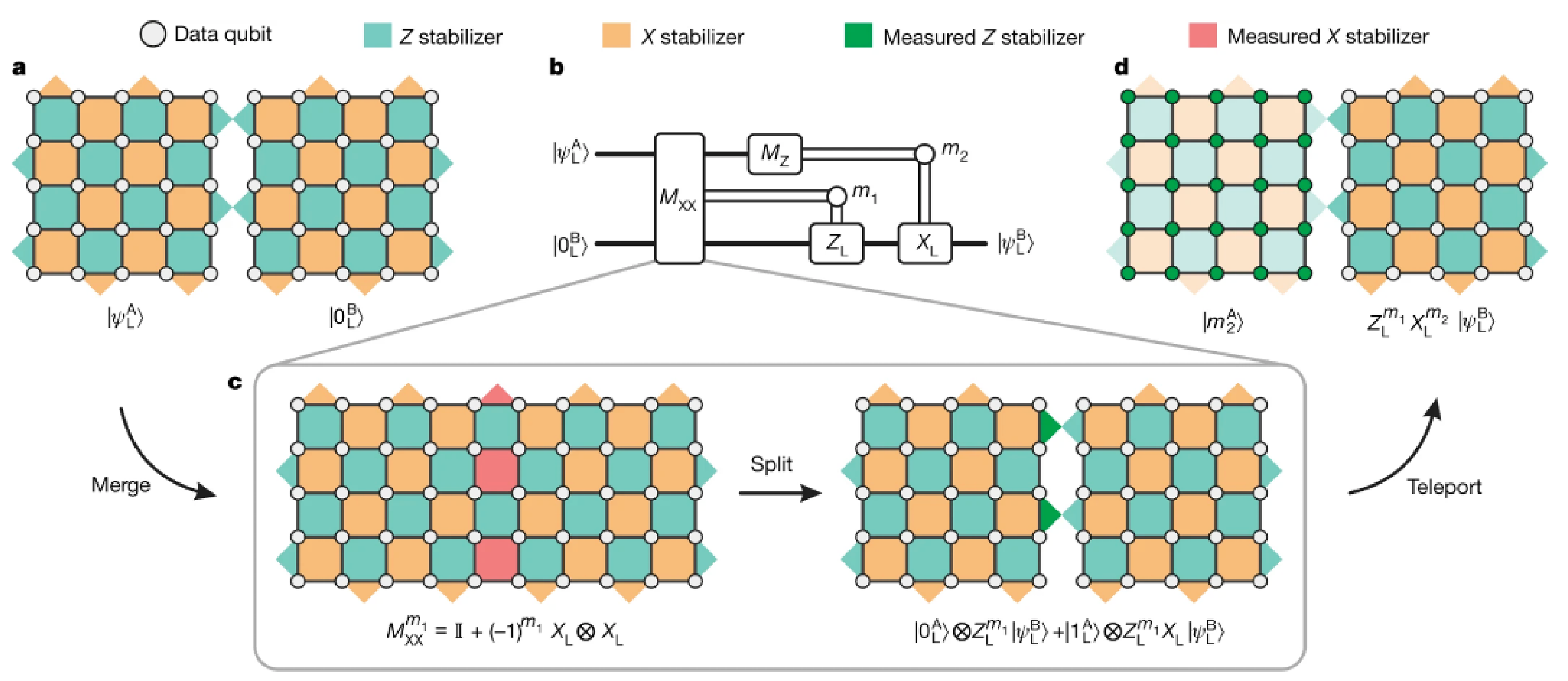
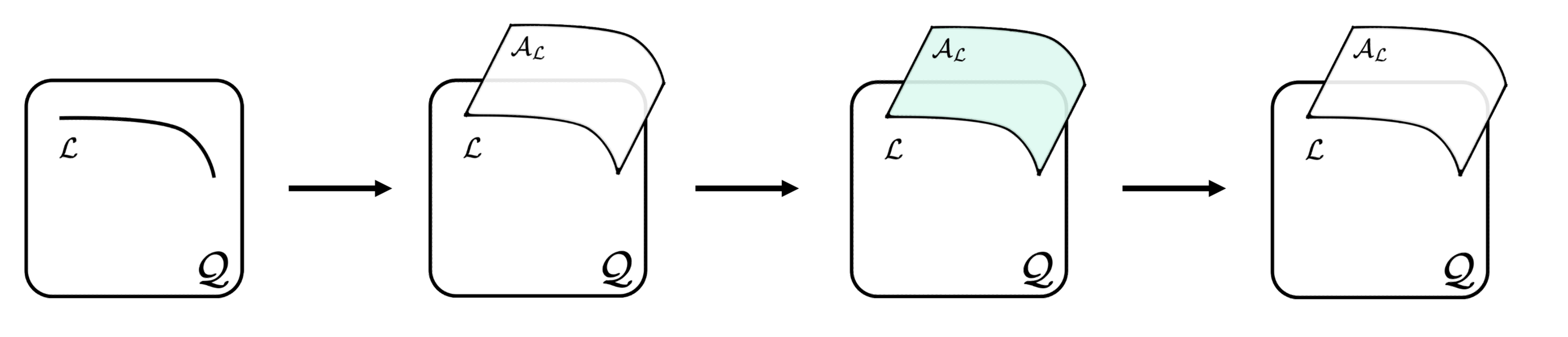


Figure 2: Illustration of surface code lattice surgery, from [Erhard et al 2020].



(a) Start: Code $\mathcal{Q}$ and operator $\mathcal{L}$. (b) Init: Initialize ancilla in product state. (c) Merge: Code deformation by measuring new stabilizers (d) Split: Measure out ancilla and return to $\mathcal{Q}$.

Figure 3: High-level illustration of the process of QLDPC code surgery, with code block $\mathcal{Q}$, logical operator $\mathcal{L}$, and ancilla system $\mathcal{A}_\mathcal{L}$.

Code Surgery with Auxilliary Graph

Let us start with an arbitrary stabilizer code $\mathcal{Q}$ and an arbitrary Pauli operator P. Let $G = (\mathcal{V}, \mathcal{E})$ be a measurement graph. We construct the measurement code as follows.

- For every edge in $\mathcal{E}$, introduce an ancilla edge qubit;
- For every vertex in $\mathcal{V}$, introduce an ancilla vertex check s_v that acts on the adjacent edge qubits by X.
- Connect vertex checks to qubits in P so that their product is P, $\prod_{v\in V} s_v = P$. Let the stabilizers $\mathcal{C}$ of $\mathcal{Q}$ act on edge qubits by Z so that all stabilizers commute.
- Choose a basis of cycles $\mathcal{U}$ of G. For every cycle, add a cycle check that acts on involved edges by Z.

By measuring all stabilizers for $O(d)$ rounds, assuming certain conditions are met (see Figure 4), we can fault-tolerantly measure P. For any code $\mathcal{Q}$, any operator of weight w, we can construct a LDPC measurement graph of size $O(w(\log w)^3)$. This is a qualitative improvement over $O(w^2)$ from prior work [8]. In practice, we can construct $O(w)$ size ancilla systems without LDPC guarantee [1, 2, 4, 7].

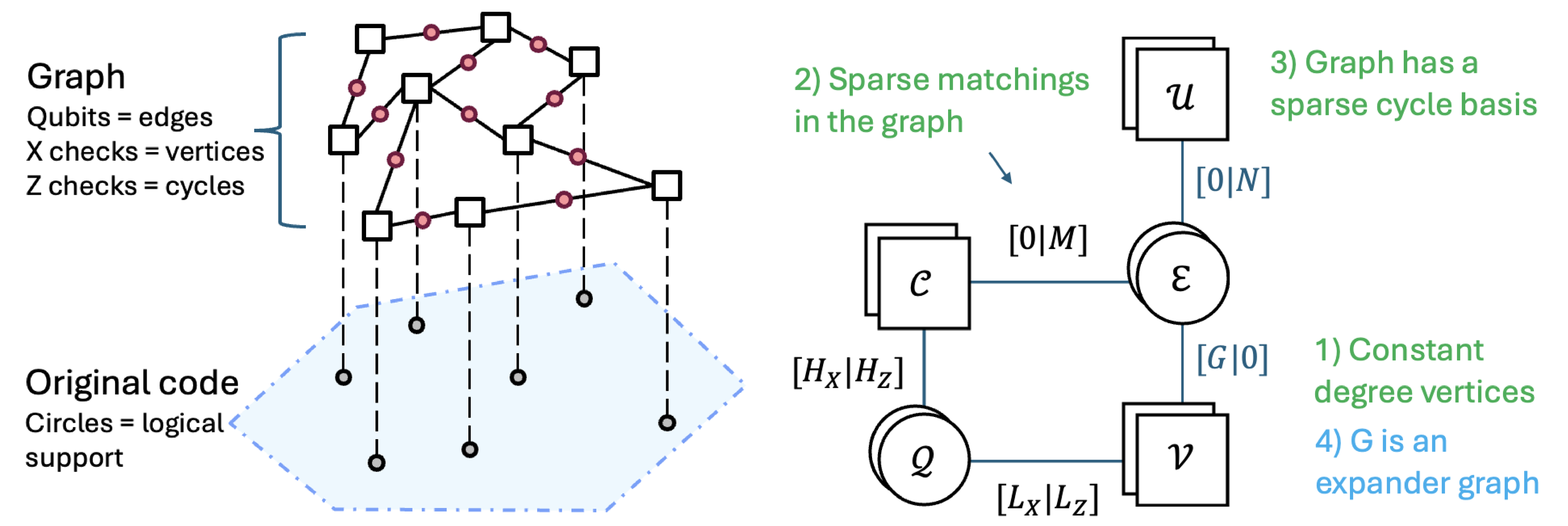


Figure 4: (Left) Graphical depiction of the ancilla system, constructed from a graph G, connected to the measured logical operator P. (Right) Tanner graph representation of the measurement code, with the code qubits and checks on the left and ancilla qubits and checks on the right. All connections are labelled by symplectic check matrices $[X|Z]$. Green conditions ensure that the measurement code is LDPC. Blue conditions ensure the measurement code has high distance.

For a detailed and thorough discussion of these methods, please see Section 3 of [6].

Case Study: Bivariate Bicycle Code

We applied our methods to construct an ancilla system on the $[[144, 12, 12]]$ BB Code. This system enables us to perform 8 different measurements. When combined with the automorphism gates of the code, we can perform the full logical Clifford group.

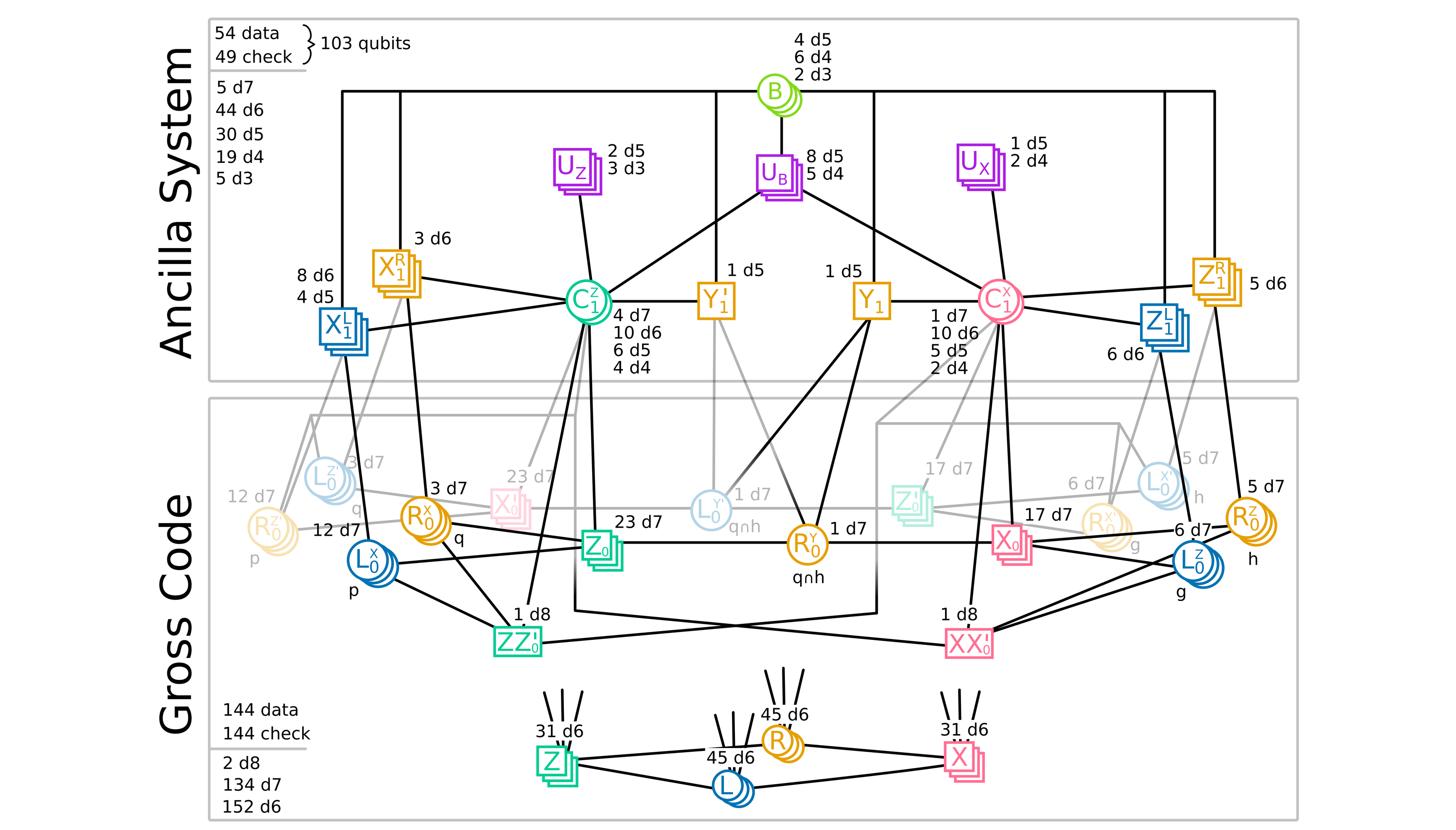


Figure 5: 103-qubit Ancilla system constructed on BB code. Circles denote qubits, boxes denote checks. The degree of qubits are displayed.

References

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